5- Motion in two dimensions

Motion in two dimensions like the motion of projectiles and satellites and the motion of charged particles in electric fields. Here we shall treat the motion in plane with constant acceleration and uniform circular motion.

5-1 Motion in two dimensions with constant acceleration

Assume that the magnitude and direction of the acceleration remain unchanged during the motion. The position vector for a particle moving in two dimensions (x, y plane) can be written as:

$$\vec{r} = x_i + y_j$$

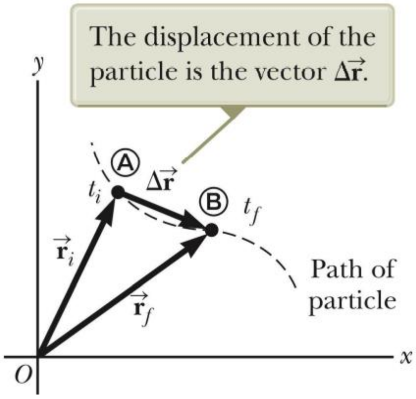


figure 5.1

Displacement In Fig. 5.1, an object is initially at position $r_i\,(t_i)$ at time ti (point A). Sometime later, t_f, the object is at position $r_f\,(t_f)$ (point B).

The displacement vector of the object is given by:

$$\Delta\vec{r} = \vec{r_f} - \vec{r_i}$$

where x, y, and r change with time as the particle moves

The velocity of the particle is given by:

$$\vec{v} = \frac{dr}{dt} = \frac{dx}{dt}\, i + \frac{dy}{dt}\, j$$

$$\vec{v} = v_x\, i + v_y\, j$$

Since the acceleration is constant then we can substitute:

$$v_x = v_{xo} + a_x t \qquad \& \qquad v_y = v_{yo} + a_y t$$

This gives:

$$v = (v_{xo} + a_x t)\, i + (v_{yo} + a_y t)\, j$$

$$= (v_{xo}\, i + v_{yo}\, j) + (a_x\, i + a_y\, j)\, t$$

Then $v = v_o + a\, t$ ---------------------------(*)

From the equation (*) we conclude that the velocity of a body at a specific time t is equal to the vector sum of the initial velocity and the velocity resulting from the uniform acceleration

Since our particle moves in two-dimensions x and y with constant acceleration then:

$$\boldsymbol{x = x_o + v_{xo}\, t + \tfrac{1}{2} a_x\, t^2} \qquad \& \qquad \boldsymbol{y = y_o + v_{yo}\, t - \tfrac{1}{2} a_y\, t^2}$$

But

$$r = x_i + y_j$$

$$r = (x_o + v_{xo}\, t + \frac{1}{2} a\, t^2)\, i + (y_o + v_{yo}\, t - \frac{1}{2}\, g\, t^2)\, j$$

$$= (x_o\, i + y_o\, j) + (v_{xo}\, i + v_{yo}\, j)\, t + \frac{1}{2}(a_x\, i + a_y\, j)\, t^2$$

$$r = r_o + v_o t + \frac{1}{2}\, a\, t^2 \text{ --- (**)}$$

From the equation (**), we conclude that the displacement vector r-r_o is the vector summation of the displacement vector resulting from the initial velocity $v_o\, t$ and the displacement resulting from the uniform acceleration $\frac{1}{2}\, a\, t^2$

5-2 Projectile motion

Projectile motion is an example of motion in two dimensions, and we will find the equations of motion for projectiles to determine the horizontal and vertical displacement, velocity, and acceleration.

A good example of the motion in two dimension it the motion of projectile. To analyze this motion let's assume that at time t=0 the projectile start at the point x_o=y_o=0 with initial velocity v_o which makes an angle θ_o, as shown in Figure 5.2

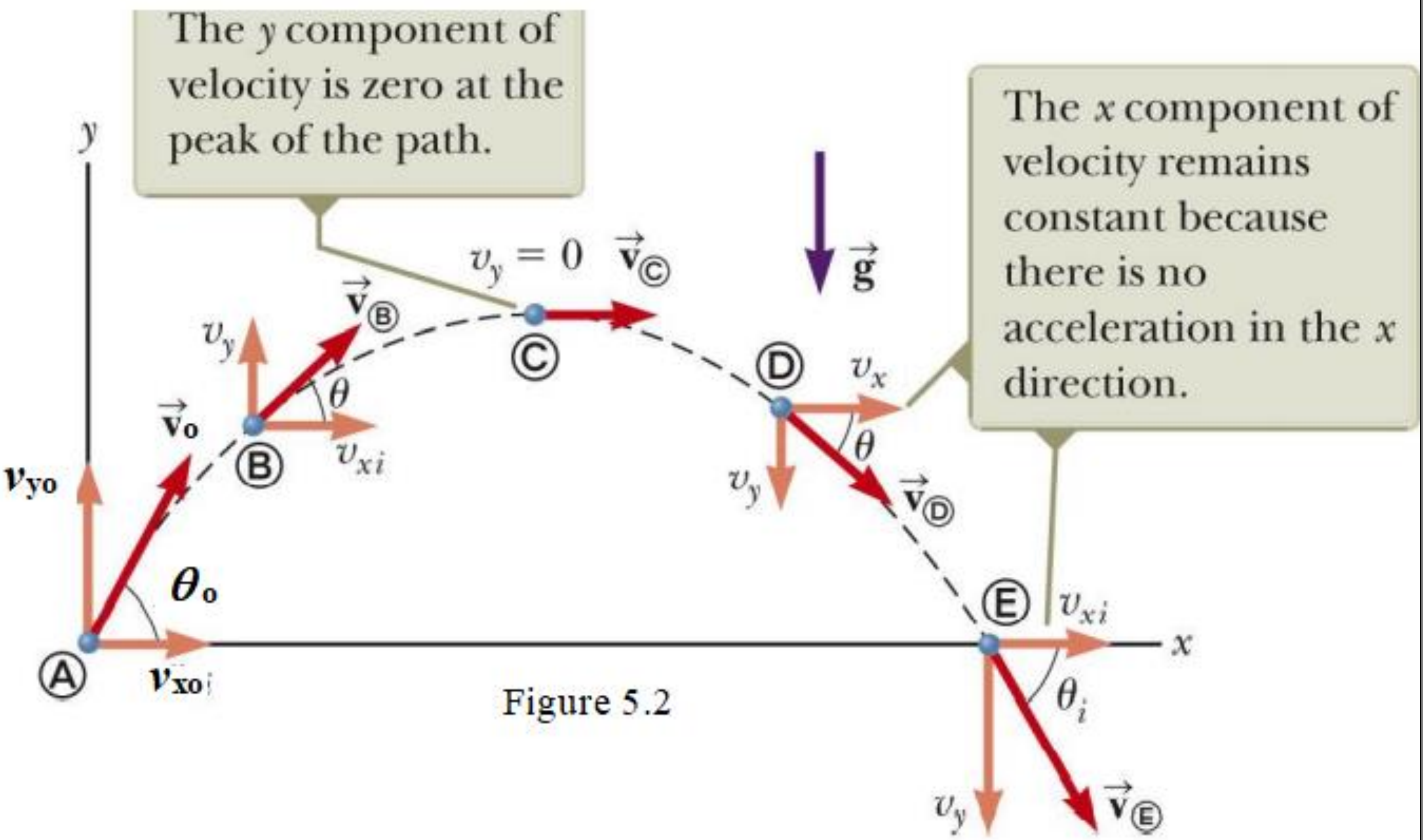


Figure 5.2

Then the two Components of Initial velocity are:

$$v_{xo} = v_o \cos\theta_o \qquad \& \qquad v_{yo} = v_o \sin\theta_o$$

The two velocity Components (v_x, v_y) at any time are:

$$v_x = v_{xo} = v_o \cos\theta_o \qquad \text{--(5-1)}$$

$$v_y = v_{yo} - gt = v_o \sin\theta_o - gt \qquad \text{--(5-2)}$$

Instantaneous position of the particle in the direction of ***x*** **is:**

$$x = v_{xo}\, t = v_o\, t \cos\theta_o \qquad \text{---(5-3)}$$

Instantaneous position of the particle in the direction of ***y is:***

$$y = v_{yo}\, t - \frac{1}{2} g\, t^2 = v_o\, t \sin\theta_o - \frac{1}{2} g\, t^2 \qquad \text{-----------------------------------(5-4)}$$

$$v = \sqrt{vx^2 + vy^2}$$

The angle

$$\theta = \tan^{-1} \frac{vy}{vx}$$

The time required for the projectile to reach its highest point, $v_y = 0$.

So, equation (5-2) $v_y = v_{yo} - gt$ *can be written:* $0 = v_{yo} - gt$

$$\therefore t = \frac{v_{y0}}{g} \qquad \Rightarrow \qquad t = \frac{v_{y_0} \sin\theta_0}{g} \qquad \text{--(5-5)}$$

When sub. Equation (5-5) in equation (5-4) get the highest point the projectile reaches:

$$y = h = \frac{v_0^2 \sin^2\theta_0}{2g}$$ show that? --(5-6)

Note that:

When calculating the total flight time, $y = 0$, ⇒ total flight time:

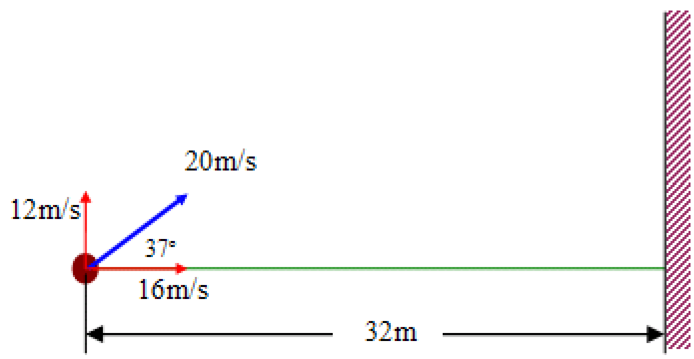
Total flight time: $\mathbf{T} = \frac{2v_0 \sin\theta_0}{g}$ show that? Hint, Using equation (5-4) ------(5-7)

Range is the horizontal distance during the flight time: $\mathbf{R} = v_{0x}\,T$

$\Rightarrow \mathbf{R} = \frac{v_0^2 \sin 2\theta_0}{g}$ Show that, where **R** projectile range. ----------(5-8)

Example

In the Figure shown below where will the ball hit the wall?



Solution

$v_x = v_{xo} = 16$m/s, $x = 32$m

Then the time of flight is given by from $T = \frac{2v_0 \sin\theta_0}{g}$

$\therefore t = 1.96\text{s} \approx 2\text{sec}$

To find the vertical height after 2s we use the relation

$y = v_{yo}\,t - \frac{1}{2} g\,t^2$, Where $v_{yo} = 12$m/s, t = 2s ⇒ $y = 4.4$m

Since y is positive value, therefore the ball hit the wall at 4.4m from the ground

To determine whether the ball is going up or down we estimate the velocity and from its direction we can know:

$v_y = v_{yo} - gt \Rightarrow v_y = -7.6$m/s

Since the final velocity is negative then the ball must be going down.