Q5. A 1.5 Kg parcel of dry air is at a temperature of 15°C and a pressure of 1013 hPa.

a. How many moles of air are in the parcel? (molecular ~~air~~ weight of air is 28.96 g/mole)

Sol.

$$\text{no. of moles } n = \frac{\text{mass}}{\text{molecular weight}} = \frac{m}{M} = \frac{1.5 \times 1000\ g}{28.96\ \frac{g}{mol}} = 51.8\ mol$$

b. What is the volume of the parcel?

Sol.

$$PV = nRT \Rightarrow V = \frac{nRT}{P} = \frac{51.8\ mol * 8.3145\ J/mol*K * (15+273)\ K}{1013 \times 100\ Pa}$$

$\therefore V = 1.22\ m^3$

c. What is the specific volume of the parcel?

Sol.

$$\alpha = \frac{V}{m} = \frac{1.22\ m^3}{1.5\ kg} = 0.8\ m^3/kg$$

d. If 50 KJ of heat are added to the parcel while its volume is held constant; what is the new temperature of the parcel?

Sol.

$$dq = C_v (T_2 - T_1) \Rightarrow dq = C_v * m (T_2 - T_1)$$

(C_v ← $\frac{C_v}{m}$)

$$50 \times 1000\ J = 717\ \frac{J}{kgK} * 1.5\ kg\ (T_2 - 288\ K)$$

$$50000\ J = 1075.5\ T_2 - 309744$$

$$\therefore T_2 = \frac{359744}{1075.5} = 334.5\ K$$

Q.6. A parcel of dry air at temperature of 15°C and pressure of 1013 hPa, heat is added to the parcel to cause it to expand. It expands at constant pressure of 1.5 times its origional Volume.

a. What is the new temperature of the parcel?

Sol.

$V_2 = 1.5 V_1$

For isobaric process, $\frac{T_2}{T_1} = \frac{V_2}{V_1} \Rightarrow \frac{T_2}{T_1} = \frac{1.5 V_1}{V_1}$

$\therefore T_2 = 1.5 T_1 \Rightarrow T_2 = 1.5 * (15+273) = 432°K = 159°C$

b. How much work (per unit mass) was done by the parcel during this expansion? [$R_d = 287.1$ J/kgK]

Sol.

$PV = R_d T \Rightarrow V_1 = \frac{R_d T_1}{P}$, $V_2 = \frac{R_d T_2}{P}$

$V_1 = \frac{287.1 * (15+273)}{1013 * 100} = 0.816$

$V_2 = \frac{287.1 * 432}{1013 * 100} = 1.22$

$W = P\Delta V = 101300 * (1.22 - 0.816) = 40925.2 \frac{J}{kg}$

$= 40.9 \frac{kJ}{kg}$

c. What was the change in specific internal energy of air parcel?

Sol.

$\Delta u = \frac{5}{2} R_d \Delta T = \frac{5}{2} * 287.1 * (432 - 288) = 103 \frac{kJ}{kg}$

d. What was the amount of heat per unit mass that was added to the air parcel?

$q = \Delta u + \Delta w = 103 + 40.9 = 143.9$ kg/kg

Q7. An air parcel is at a temperature of 15°C and a pressure of 1013 mb. Heat is added to the parcel to cause it to expand. It expands at constant temperature until its volume is 1.5 time its origional volume.

a. What is the new pressure of the air parcel?

Sol.

$V_2 = 1.5\,V_1$, for isothermal process

$P_1V_1 = P_2V_2$

$P_1 V_1 = P_2 * 1.5 V_1$

$\therefore P_1 = 1.5\,P_2$

$P_2 = \frac{P_1}{1.5} = \frac{1013\ hPa}{1.5} = 675.3\ hPa$

b. How much heat per unit mass was added to the air parcel, $R' = 287.1\ J/kg.K$

Sol.

For isothermal processes, $q = -R'T \ln \frac{P_f}{P_i}$

$\therefore q = 287.1 * (15+273) * \ln \frac{675.5}{1013}$

$= 33.5\ kJ/kg$

c. How much work per unit mass was done in expanding the air parcel?

Sol.

In isothermal process, $dq = dw$

$\therefore dw = -33.5\ kJ/kg$

The negative sign (−) was put because the system did work on surrounding.

d. What was the change in specific internal energy of the air parcel?

Sol. Δu = Zero. because u is a function of temperature, and because temperature is constant then Δu = zero.

Q8. A dry air parcel at an initial temperature of 20°C and a pressure of 950 mb is forced to rise adiabatically up a mountain slope. The top of the mountain is at a pressure of 720 mb.

a. What is the temperature of the air parcel when it reaches the top of the mountain?

Sol.

$\because TP^{(1-\gamma)/\gamma} = \text{constant}$, $\gamma = 1.4$

$\therefore T_1 P_1^{(1-\gamma)/\gamma} = T_2 P_2^{(1-\gamma)/\gamma}$

$\therefore T_2 = \dfrac{T_1 P_1^{(1-\gamma)/\gamma}}{P_2^{(1-\gamma)/\gamma}} = \dfrac{(20+273) * (950 \times 100 hPa)^{(1-1.4)/1.4}}{(720 \times 100)^{(1-1.4)/1.4}}$

$= \dfrac{293 \times (95000)^{-0.2857}}{(72000)^{-0.2857}} = \dfrac{11}{0.04}$

$= 275K \simeq 2°C$

b. What is the work done by the air parcel

Sol.

$\because dw = du$ ⟶ for dry adiabatic process ($dq = 0$)

$dw = C_v dT$

$= 718 \times ((2+273) - (20+273))$

$= 718 \times (275 - 293)$

$= 718 \times (-18) = -12924$ J/kg

$= \dfrac{-12924}{1000} = -12.924 \dfrac{kJ}{kg}$