2. The Sun as a Radiation Source

All matter emits electromagnetic radiation at all wavelengths due to its atomic and molecular agitation unless it is at a temperature of 0 K. The intensity and the spectral distribution of the radiation is solely determined by the temperature and the material properties of the emitting body (particularly of its surface). Both are described by the famous radiation laws of Kirchhoff and Planck.

A good approximation of intensity and spectral distribution of the radiation of a given body can be made by assuming it to be a *blackbody* or a *blackbody radiator*. A perfect blackbody is an object that does not reflect any radiation whatever, but absorbs all radiation incident upon it. It emits the maximum amount of energy at each wavelength and into all directions.

2.1 Radiation Laws

Planck's law for the spectral radiance describes the energy distribution of radiation emitted by a blackbody at temperature T into a unit solid angle as a function of wavelength:

$$L_\lambda(T) = \frac{2hc^2}{\lambda^5}\frac{1}{e^{hc/\lambda kT}-1} \quad [\mathrm{Wm^{-2}sr^{-1}\mu m^{-1}}], \tag{2.1}$$

or in terms of frequency:

$$L_\nu(T) = \frac{2h\nu^3}{c^2}\frac{1}{e^{h\nu/kT}-1} \quad [\mathrm{Wm^{-2}sr^{-1}s}], \tag{2.2}$$

where $k = 1.381 \cdot 10^{-23}$ WsK^{-1} is Boltzmann's constant, $h = 6.626 \cdot 10^{-34}$ Ws^2 is Planck's constant and $c = 2.998 \cdot 10^8$ ms^{-1} is the vacuum velocity of light.

The hemispherical spectral radiant flux emerging from a unit surface (radiant exitance) then is (Fig. 2.1):

$$M_\lambda(T) = \pi L_\lambda(T) \quad [\mathrm{Wm^{-2}\mu m^{-1}}] \tag{2.3}$$

For each temperature the blackbody emission appoaches zero for very small and very large wavelengths. The curve for a warm blackbody lies above the curve for a cooler blackbody at each wavelength.

Approximately, the sun (more strictly, its gaseous surface) is a blackbody emitter. The effective surface temperature is $\sim 5780\,\mathrm{K}$.

The relationship between the wavelength of a blackbody's maximum emission λ_m and the corresponding absolute temperature is given by ***Wien's displacement law***:

$$\lambda_m = \frac{2897 \mu\mathrm{mK}}{T}. \tag{2.4}$$

2.2 Radiant Flux Emitted by the Sun

Integration of Planck's function (Eq. 2.1) over the entire wavelength domain leads to the fundamental ***Stefan-Boltzmann law***, which gives the total radiant flux density emitted by a blackbody at temperature T (emitted radiation per unit time and area):

$$M = \int_0^\infty M_\lambda(T)\,d\lambda = \pi \int_0^\infty L_\lambda(T)\,d\lambda = \sigma T^4 \tag{2.5}$$

where $\sigma = 2\pi^5 k^4/15c^2h^3 = 5.67 \cdot 10^{-8}\,\mathrm{Wm^{-2}K^{-4}}$ is the Stefan-Boltzmann constant.

Inserting $T = 5780\,\mathrm{K}$ yields the radiant exitance from the sun's surface:

$$\begin{aligned} M_{sun} &= 5.67 \cdot 10^{-8} \cdot 5780^4\ \mathrm{Wm^{-2}} \\ &= 6.33 \cdot 10^7\ \mathrm{Wm^{-2}}. \end{aligned} \tag{2.6}$$

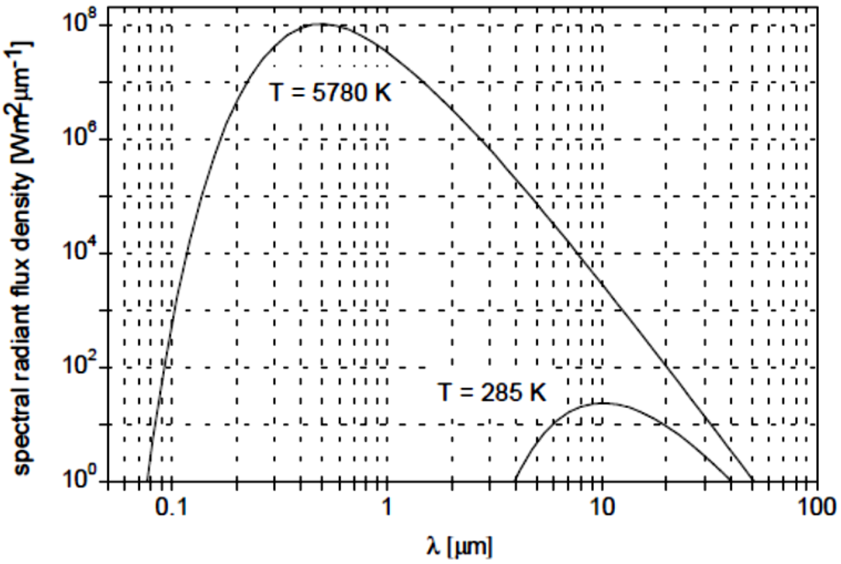

Fig. 2.1. Spectra of emitted blackbody radiation at $T = 5780\,\mathrm{K}$ (sun surface) and $T = 285\,\mathrm{K}$ (earth surface).

With $A_{sun} = 4\pi r_{sun}^2$ and $r_{sun} = 6.96 \cdot 10^8$ m, the total radiant flux emitted by the sun is:

$$\begin{aligned} \Phi_{sun} &= M_{sun} \cdot A_{sun} \\ &= 6.33 \cdot 10^7 \text{ Wm}^{-2} \cdot 4 \cdot 3.14 \cdot (6.96 \cdot 10^8 \text{ m})^2 \\ &= 3.85 \cdot 10^{26} \text{ W}. \end{aligned} \tag{2.7}$$

2.3 Solar Constant

From energy conservation principles the total radiant flux through the surface of the sun (Φ_{sun}) equals the flux through any spherical surface concentric to the sun. Especially, for a sphere with a radius $\bar{r}_{ES}$, the mean distance between sun and earth, it is (Fig. 2.2): [1]

$$4\pi r_{sun}^2 M_{sun} = 4\pi \bar{r}_{ES}^2 E_{sc} \tag{2.8}$$

The ratio $f_s = (r_{sun}/\bar{r}_{ES})^2 = 2.165 \cdot 10^{-5} \simeq 1/46200$ is called ***dilution factor*** and is an often used number in sun-earth astronomy.

From this, the solar radiant flux passing through a unit area at the mean distance of the earth from the sun is:

$$E_{sc} = f_s M_{sun} = 1367 \text{ Wm}^{-2} \pm 4 \text{ Wm}^{-2}. \tag{2.9}$$

E_{sc} is called the ***solar constant***. This value has changed in the past as new measurement techniques (e.g. satellites) have been applied. In the literature a value of 1353 Wm^{-2} is given frequently. This value is out of date.

The solar constant is the solar radiant flux received on a surface of unit area perpendicular to the sun's direction at the average sun-earth distance outside the earth's atmosphere.

2.4 Total Solar Radiant Flux Received by the Earth

Being A_e the cross sectional area of the earth disk as seen by the sun and r_e = 6371 km the mean earth radius, it is:

$$\begin{aligned} \Phi_e &= A_e E_{sc} = \pi r_e^2 E_{sc} \\ &= 3.142 \cdot (6.371 \cdot 10^6 \text{ m})^2 \cdot 1367 \text{ Wm}^{-2} = 1.75 \cdot 10^{17} \text{ W}. \end{aligned} \tag{2.10}$$

Φ_e is the mean total radiant flux the earth receives from the sun. From this, the total solar energy received by the earth per year is:

$$Q_e = 5.51 \cdot 10^{24} \text{ J} = 1.53 \cdot 10^{18} \text{ kWh}. \tag{2.11}$$

Comparing this number with the annual world primary energy consumption in 1997 [2], $10.2 \cdot 10^{13}$ kWh, results in a factor of $\sim$ 15000.

[1] $\bar{r}_{ES}$ is called an astronomic unit: 1 A.U. $\simeq$ 149.6 million km.

[2] BP Statistical Review of World Energy, 1998 (http://www.bp.com/bpstats)

2.5 Extraterrestrial Radiation

The orbit of the earth around the sun is slightly elliptical with the sun at one of the foci and therefore causes a change of the sun-earth distance r_{ES} throughout the year. This variation is expressed by the eccentricity correction factor ϵ_0 (Spencer, 1971):

$$\epsilon_0 = \left(\frac{\bar{r}_{ES}}{r_{ES}}\right)^2$$
$$\simeq 1.00011 + 0.034221 \cos d + 0.00128 \sin d \qquad (2.12)$$
$$+0.000719 \cos 2d + 0.000077 \sin 2d, \qquad (2.13)$$

where $d = 2\pi(n-1)/365$ is the day angle in radians and n is the number of the day in the year (n = 1 on 1 January).
In most applications the simpler approximation

$$\epsilon_0 \simeq 1 + 0.033 \cdot \cos\left(\frac{360 \cdot n}{365}\right) \qquad (2.14)$$

can be used.
With this, the extraterrestrial radiation at normal incidence is given by:

$$G_{on} = \epsilon_0 G_{sc}$$
$$\simeq G_{sc}\left[1 + 0.033 \cdot \cos\left(\frac{360 \cdot n}{365}\right)\right]. \qquad (2.15)$$

Fig. 2.3 gives the annual variation according to Eq. 2.15.

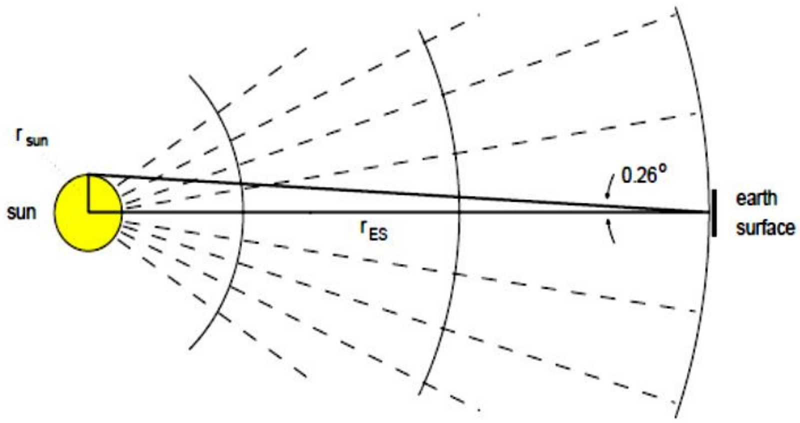

Fig. 2.2. Sun-Earth geometry.

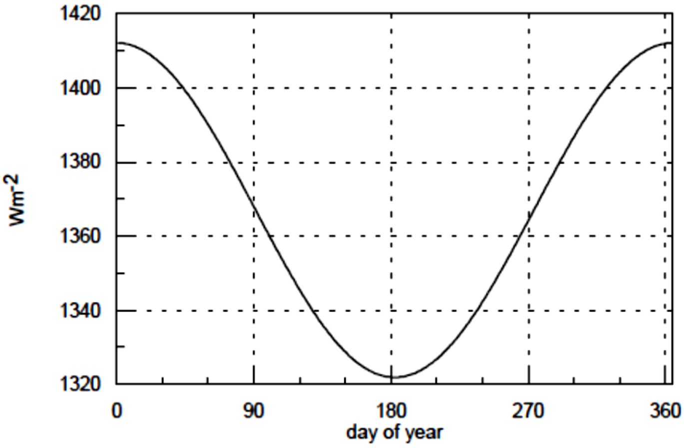

Fig. 2.3. Annual variation of the extraterrestrial irradiance at normal incidence due to the varying sun-earth distance.