**13. Uniform Continuity**

(13.1)**Definition**: If be metric spaces. We said that a function is an uniform continuous on , if , then .

(13.2)**Theorem**: Every uniform continuous is continuous.

**Proof:** let are metric spaces and let a function is an uniform continuous. Let , we must prove that be continuous at .

Let since is an uniform continuous , then

, since is a continuous at is a continuous.

(13.3)**Example:** Let be usual metric space and a function defined by , then is continuous, but does not uniform continuous.

**Solution:**  and

Let (by Archimedes property)

Put but

does not uniform continuous.

**Real- Valued Functions**

(13.4)**Definition:** Let . Define as following:

(13.5)**Theorem:** If which denoted to set of all a continuous functions and defined from into and , then

1. .
2. .
3. .
4. .
5. .

**Proof:** (1) let

Since are continuous functions

are continuous at

Since is continuous at

Since is continuous at

Put min

is continuous at .

**Boundedness**

(13.6)**Definition:** Let be metric space and . We said that is bounded in , if sup or is bounded in . We say that is bounded space, if .

(13.7)**Theorem:** Let be metric space and . We said that is bounded in .

(13.8)**Example:** In usual metric space , we have

1. be a bounded, since .
2. A space is unbounded, since .

(13.9)**Definition:** Let be metric space and is usual metric space. We said that a function is a bounded, if .

**Intermediate Value Property**

(13.10)**Definition**: Let is usual metric space. We said that satisfies an intermediate value property, if between between .

(13.11)**Example:** Let be usual metric space and let a function defined by , then a function satisfies an intermediate value property.

**Solution:** let and let

Since

Since satisfies an intermediate value property.

(13.12) **Theorem** (**Intermediate Value Theorem**)

Let is usual metric space. If a function is a continuous, then between in .

(13.13)**Example:** Let be usual metric space. If a function defined as , then satisfies an intermediate value property, but its discontinuous.