**10. Convergence**

(10.1) **Definition**: Let be a sequence in a metric space . We said that is a convergent in , if .

or where

This means where .

(10.2)**Theorem:** If is a convergent in , then the convergence point is a unique.

**Proof:** let and such that .

Let

Since

Since

Put max .

, but this is a contradiction .

(10.3)**Example:** Let be a sequence in , since is a convergent

Let

Since is an usual metric space

This means is convergent in , if with the center.

(10.4)**Theorem:** Let is a subset in , then in .

(10.5) **Definition**: Let be a sequence in a metric space . We said that is Cauchy sequence in , if .

(10.6)**Theorem:** Every convergent sequence in a metric space be Cauchy sequence.

**Proof:** let be a convergent sequence in

Let , since

If

is Cauchy sequence.

(10.7)**Note:** Not necessary that every Cauchy sequence in a metric space is a convergent, for example.

(10.8)**Example:** Let , a function defined by .

**Solution:** Let , we note that is a metric space and be Cauchy sequence in , but does not convergent to .

(10.9)**Theorem:** Let a metric space and in , then .

**Proof:**

.

(10.10)**Example:** Let is a discrete metric space and in . Prove that .

**Proof:** let .

Since .

(10.11)**Definition:** We said that is complete, if for all Cauchy sequence is a convergent.

(10.12)**Example:** Euclidean space be complete metric space.

**Solution:** let

Let be Cauchy sequence in .

Let

Cauchy sequence in

Since is complete field

Put

So, be convergent in .