٢٠٢٠ - ٢٠٢١ الامتحانات الثانية
الحساب
y=5

Q_1: In $\mathbb{Z}$, prove that: $[x,y]=[0,a]$, for some $a\in\mathbb{N}$, if $x<y$.

proof: By def. of $<$ on $\mathbb{N}$, $\exists\ a\in\mathbb{N}$ such that

$y=x+a \rightarrow x+a=y \rightarrow x+a=y+0$

$\leftrightarrow (x,y)R^*(0,a)$ (Def. of R^* on $\mathbb{N}$)

$\leftrightarrow [x,y]=[0,a]$.

Q_2: Let $a\in\mathbb{N}$. For each $b\in\mathbb{N}$ such that $a\neq b$ and $b\nless a$, prove that $a<b$.

proof: By Trichotomy Theorem, for each $n,m\in\mathbb{N}$, one of the following is true

(a) $n=m$ or (b) $n<m$ or (c) $m<n$.

since $a\neq b$, so (a) is faild
since $b\nless a$, so either (b) held or (c). Therefore,

$a<b$.

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Q3: Write the law of multiplication on $\mathbb{N}$, and then compute $((2\cdot 5^+)^+\cdot 7^+)^+\cdot 5$.

Sol:

$$\cdot(a,b) = a\cdot b = \begin{cases} a\cdot 0 = 0 & \text{if } b=0 \\ a\cdot c^+ = a + ac & \text{if } b\neq 0 \ (b=c^+) \end{cases}$$

$$2\cdot 5^+ = 2+2\cdot 5 = 2+10 = 2+9^+ = (2+9)^+ = 11^+ = 12$$

$$(2\cdot 5^+)^+ = 12^+ = 13$$

$$(2\cdot 5^+)^+\cdot 7^+ = 13\cdot 7^+ = 13+13\cdot 7 = 13+91$$
$$= 13+90^+ = (13+90)^+$$
$$= 103^+ = 104$$

$$((2\cdot 5^+)^+\cdot 7^+)^+ = 104^+ = 105$$

$$((2\cdot 5^+)^+\cdot 7^+)^+\cdot 5 = 105\cdot 5 = 105\cdot 4^+ = 105+105\cdot 4$$
$$= 105+420 = 105+419^+$$
$$= (105+419)^+ = 524^+ = 525$$

Q4: Assume the function $g: \mathbb{N} \to \mathbb{Z}$, is defined by $g(c) = [c,0]$. Then prove that
$g(a\cdot(n+m)) = g(an) + g(am)$, and then find
$g(k+4)\cdot g(5)$ in details.

Sol:

$$g(a\cdot(n+m)) = g(an+am) \quad (\text{since } a,n,m \in \mathbb{N})$$
$$= [an+am, 0] \ \text{— (1)}$$

$$g(an)+g(am) = [an,0]+[am,0] = [an+am, 0+0]$$
$$= [an+am, 0] \ \text{— (2)}$$

from (1) and (2) The prove is finshed.

$$g(k+4) = [k+4, 0] \wedge g(5) = [5,0]$$

$$g(k+4)\cdot g(5) = [k+4, 0]\cdot[5,0] = [(k+4)\cdot 5 + 0\cdot 0, (k+4)\cdot 0 + 0\cdot 5]$$
$$= [5k+20, 0]$$

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Q5: (i) If $1 = \{\phi\}$, then find 4.

(ii) In $\mathbb{Z}$, show that $([10,8]+[2,3]) < [6,2]$.

Sol (i) $1 = \{\phi\}$, $2 = \{0,1\} = \{\phi, \{\phi\}\}$.

$3 = \{0,1,2\} = \{\phi, \{\phi\}, \{\phi, \{\phi\}\}\}$.

$4 = \{0,1,2,3\} = \{\phi, \{\phi\}, \{\phi, \{\phi\}\}, \{\phi, \{\phi\}, \{\phi, \{\phi\}\}\}\}$.

(ii) $[10,8] + [2,3] = [10+2, 8+3] = [10+1^+, 8+2^+]$
$= [(10+1)^+, (8+2)^+] = [11^+, 10^+] = [12,11]$

Now To show that $[12,11] < [6,2]$.

$[r,t] < [u,v] \Leftrightarrow r+v < t+u$.

$12+2 = 12+1^+ = (12+1)^+ = 13^+ = 14$.
$11+6 = 11+5^+ = (11+5)^+ = 16^+ = 17$

and $17 = 14+3$ $\therefore$ By def. of $<$ on $\mathbb{N}$

$14 < 17 \Leftrightarrow [12,11] < [6,2]$.

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