Lecture 2: Precipitation (Part 1)

2.1 Introduction

Water may take a number of different forms in the atmosphere. These forms are collectively termed 'precipitation', which includes rain, drizzle, sleet (partly melted snowflakes, or rain and snow falling together), snow and hail. The intensity and duration of precipitation are extremely variable in most areas of the world. The source of precipitation is water vapor, which is always present in the atmosphere in varying amounts, although it makes up less than 1% by volume. However, the water vapor in the air must be cooled to allow water to be condensed into cloud droplets. These droplets then grow to form precipitation particles. The mass of water in the atmosphere in both liquid and vapor forms is around 1.3×10^{16} kg, compared with the mass of water in the oceans of around 1.3×10^{21} kg. Nevertheless this water is distributed very unevenly, and is transported by the circulation of the atmosphere. We will consider the basic thermodynamic processes which lead to the formation of precipitation, and describe the atmospheric systems within which these processes occur.

2.2 Equation of state for a perfect gas

A perfect gas (ideal gas) obeys the physical laws of Boyle and Charles. The gas equation (or equation of state) is:

$$p = \rho R T \qquad (2.1)$$

where $R = 287\ J\ kg^{-1}K^{-1}$ is known as the universal gas constant.

$$\text{Boyle's law:} \qquad p_1 V_1 = p_2 V_2 \qquad at\ constant\ T$$

$$\text{Charles' law:} \qquad \frac{p_1}{T_1} = \frac{p_2}{T_2} \qquad at\ constant\ V$$

where V is volume. Combining the two laws we get:

$$\frac{p_1 V_1}{T_1} = \frac{p_2 V_2}{T_2} = C$$

C is constant depends on the mass of gas ($=287\ J\ kg^{-1}K^{-1}$ specific gas constant). These laws, are true for a perfect gas, and are nearly true for atmospheric gases.

Dalton's law states that in a mixture of gases, the total pressure is equal to the sum of the pressures which would be exerted by each gas if it filled the volume under consideration at the same temperature. Hence a mixture of gases, such as the atmosphere, behaves like a single gas provided the mixture gas constant R is given by:

$$R = \frac{\sum m_i R_i}{\sum m_i} \qquad (2.2)$$

where m_i and R_i is the molecular weight and gas constant of the ith gas.

2.3 First law of thermodynamics

The first law of thermodynamics is derived from two facts: (i) heat is a form of energy; and (ii) energy is conserved. Conservation of energy Q may be expressed by

$$dQ = dU + dW \qquad (2.3)$$

where U is the internal energy and W is the work done by the gas in moving from one state to another. Generally, terms of unit mass of gas:

$$dq = du + dw \qquad (2.4)$$
$$dq = C_v \, dT + p \, d\alpha \qquad (2.5)$$

$\alpha = \dfrac{1}{\rho}$ is specific volume. Differentiation of the equation of state $(p\alpha = RT)$ gives:

$$p \, d\alpha + \alpha \, dp = R \, dT$$

$$p \, d\alpha = R \, dT - \alpha \, dp \qquad (2.6)$$

Substitute (2.6) in (2.5) we get:

$$dq = C_v \, dT + R \, dT - \alpha \, dp$$

$$dq = (C_v + R) \, dT - \alpha \, dp$$

$$dq = C_p \, dT - \alpha \, dp \qquad (2.7)$$

where $C_p = R + C_v = 1004 \, Jkg^{-1}K^{-1}$ and $C_v = 717 \, Jkg^{-1}K^{-1}$ are the specific heat at constants pressure and volume, respectively.

Both of equations (2.5) and (2.7) represent the first law of thermodynamics.

2.4 Atmospheric processes: dry adiabatic lapse rate

It is assumed that $dq=0$ for most air parcel movements. This assumption can be made whenever the motion is fast so that the heat exchange between the parcel and the surroundings is negligible. (*Why?*)

For adiabatic motion, equations of first law of thermodynamics become:

$$C_v \, dT + p \, d\alpha = 0 \qquad (2.8)$$
$$C_p \, dT - \alpha \, dp = 0 \qquad (2.9)$$

Solving for p in equation (2.9) from the equation of state $(p = \dfrac{1}{\alpha} R \, T)$

$$C_v \, dT + \frac{1}{\alpha} R \, T \, d\alpha = 0$$

$$C_v \, dT = -R \, T \frac{d\alpha}{\alpha}$$

$$C_v \frac{dT}{T} = -R \, \frac{d\alpha}{\alpha}$$

$$\int_{T_1}^{T} \frac{dT}{T} = -\frac{R}{C_v} \int_{\alpha_1}^{\alpha} \frac{d\alpha}{\alpha}$$

$$\ln \, (T - T_1) = -\frac{R}{C_v} (\ln(\alpha - \alpha_1))$$

Now, by taking the exponential (e) for the two sides:

$$\frac{T}{T_1} = \left(\frac{\alpha}{\alpha_1}\right)^{-\frac{R}{C_v}} \qquad (2.10)$$

If T increases, α will decrease and vice versa.

From equation of state: $\qquad \alpha = \frac{R\,T}{p} \qquad (2.11)$

Substitute (2.11) in (2.10) and using (2.9) we get:

$$C_p\,dT - \frac{R\,T}{p}\,dp$$

$$\int_{T_1}^{T} \frac{dT}{T} = -\frac{R}{C_v} \int_{p_1}^{p} \frac{dp}{p}$$

$$\ln\frac{T}{T_1} = \frac{R}{C_P} \ln\frac{p}{p_1}$$

$$\frac{T}{T_1} = \left(\frac{p}{p_1}\right)^{\frac{R}{C_P}} \qquad (2.12)$$

If T increases P will increases and vice versa.
From equation (2.10) an equation (2.11) we get:

$$\left(\frac{\alpha}{\alpha_1}\right)^{-\frac{R}{C_v}} = \left(\frac{p}{p_1}\right)^{\frac{R}{C_P}}$$

$$\frac{\alpha}{\alpha_1} = \left(\frac{p}{p_1}\right)^{-\frac{C_v}{C_P}}$$

An increase in p corresponds to a decrease in α and vice versa
From equation 2.12 and taking a reference pressure of 1000 millibars, we may define potential temperature θ as follows:

$$\theta = T \left(\frac{1000}{p}\right)^{K} \qquad (2.13)$$

Where $K = R/C_p$. Therefore, θ is a constant for a dry adiabatic process, that is, one with no condensation or evaporation. Knowing that the hydrostatic equation is:

$$\frac{\partial p}{\partial z} = \frac{-g}{\alpha} \qquad (2.14)$$

From equation (2.9), and differentiating with respect to height z, and using (2.14):

$$C_p \frac{dT}{dz} - \alpha \frac{dp}{dz} = 0$$

$$C_p \frac{dT}{dz} = -g$$

$$\frac{dT}{dz} = -g/C_p = \gamma_d = -9.76\ ^oC\ km^{-1} \qquad (2.15)$$

Therefore if a parcel of air rises dry adiabatically, its temperature will fall at the rate of about 10°C km^{-1}. The quantity γ_d is the *dry adiabatic lapse rate* (DALR).

2.5 The Clausius-Clapeyron Equation

Clausius-Clapeyron equation calculates the change of the saturation vapor pressure with temperature (de_s/dT) during a phase change.

$$\frac{de_s}{dT} = \frac{L_{12}}{T(\alpha_2 - \alpha_1)} \qquad (2.16)$$

If L_{12}, α_1 and α_2 are known functions of T, then (2.16) can be integrated to obtain a relation between saturation vapor pressure and T.

Since $\alpha_2 \gg \alpha_1$ (specific volume of water vapor is much greater than specific volume of liquid water or ice), equation 2.16 becomes:

$$\frac{de_s}{dT} = \frac{L_{12}}{T\alpha_2} \qquad (2.17)$$

From the equation of state ($\alpha_2 = R_v T/e_s$) we get:

$$\frac{de_s}{e_s} = \frac{L_{12}}{R_v}\frac{dT}{T^2} \qquad (2.18)$$

Therefore,

$$\ln\left(\frac{e_s}{e_{s0}}\right) = \frac{L_{12}}{R_v}\left(\frac{1}{T_0} - \frac{1}{T}\right) \qquad (2.19)$$

Where e_{s0} is the saturation vapor pressure at T_0.

For evaporation, $e_{s0} = 6.11$ mb and $T_0 = 273\ K$, which implies mb and To=273 K, which implies

$$\ln\left(\frac{e_s}{6.11\ mb}\right) = \frac{L_{evap}}{R_v}\left(\frac{1}{273} - \frac{1}{T}\right) \qquad (2.20)$$

$$\text{or} \qquad e_s = 6.11\ mb\ \exp[\frac{L_{evap}}{R_v}\left(\frac{1}{273} - \frac{1}{T}\right)] \qquad (2.21)$$

For sublimation,

$$\ln\left(\frac{e_s}{6.11\ mb}\right) = \frac{L_{sub}}{R_v}\left(\frac{1}{273} - \frac{1}{T}\right) \qquad (2.22)$$

$$\text{or} \qquad e_s = 6.11\ mb\ \exp[\frac{L_{sub}}{R_v}\left(\frac{1}{273} - \frac{1}{T}\right)] \qquad (2.23)$$

Ex: Calculate the change in the melting point of ice if the pressure is increased from 1 to 2 atm, given that: $\alpha_i = 1.0908 \times 10^{-3}\ m^3 kg^{-1}$, $\alpha_w = 1.001 \times 10^{-3}\ m^3 kg^{-1}$

$$L_{melt} = 3.34 \times 10^5\ Jkg^{-1} \quad at\ 0^oC$$

Solution: Using Clausius-Clapeyron equation,

$$dT = T(\alpha_w - \alpha_i)\frac{dp}{L_{melt}}$$

$$= 273 \times (1.001 - 1.0908) \times 1.013 \times 10^5/3.34 \times 10^5\ = -0.00744\ deg$$

Therefore, an increase in pressure of 1 atm decreases the melting point of ice by about 0.007 deg. Usually, the melting point increases with increasing pressure. But ice is unusual because $\alpha_w < \alpha_i$.

Equations (2.20) and (2.22) plus a similar equation for melting allow us to plot the curves of saturation pressure versus temperature. These curves describe the *T-e* relations during phase changes ($e=e_s$).
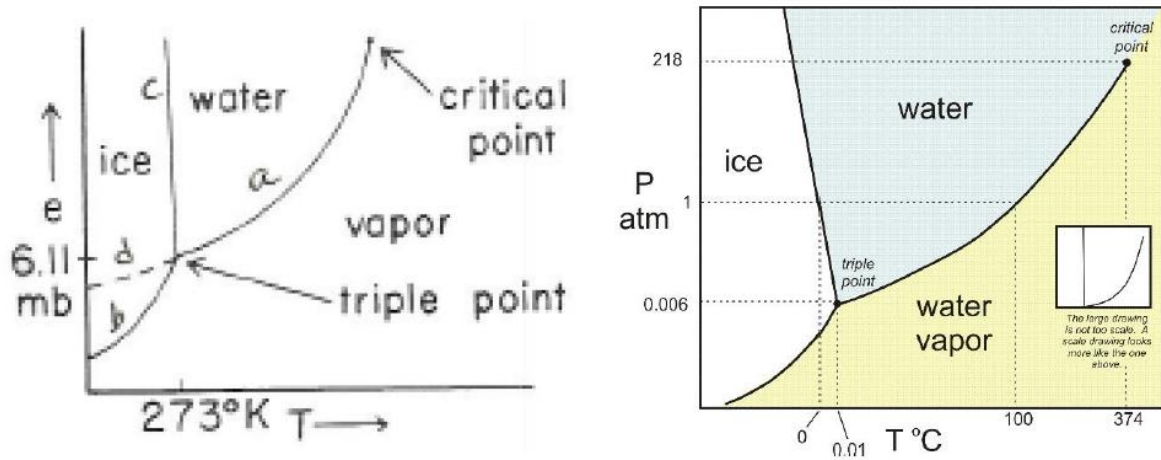

Figure 2.1 *T, e* phase diagram for water substance.

All curves begin at the triple point.

(a) Evaporation curve a: curves upward exponentially to the right according to Eq. (2.20) ($T > 273$ K, and $e_s > 6.11$ mb). Along this curve, water and vapor are in equilibrium until it reaches the critical point ($T = 374$ K) where only vapor can exist. At some point along this curve, water starts to boil. That is the point when $e_s = p_{atm}$.

(b) Sublimation curve b: curves downward exponentially to the left of the triple point ($T < 273$ K, $e_s < 6.11$ mb). This curve is steeper than the evaporation curve because $L_{sub} > L_{evap}$.

(c) Melting curve c: since $\alpha_w - \alpha_i$ is almost zero (a very small negative number), de_s/dT is almost $-\infty$, based on the Clausius-Clapyeron equation. Thus the curve is almost vertical, but tilts very slightly to the left.

(d) Supercooled water d: When water which does not freeze below 273 K (0°C), it is called supercooled water. For supercooled water, within a certain temperature range of $T < 0$°C, $e_{sw} > e_{si}$ (curve d).

Therefore, in a cloud with mixture of ice crystals and supercooled water (cloud) droplets, water vapor will first condense on the ice because e_{si} is smaller than e_{sw}.

In other words, in the competition for water vapor, ice will win over supercooled water. This leads to the well-known Bergeron-Findeison process. That is, ice crystal in a cloud of supercooled water droplets will grow faster than the water droplets.

2.6 Atmospheric processes: saturated adiabatic lapse rate

For a sample of moist air in which no evaporation or condensation occurs, equation 2.16 may be used. However, when condensation occurs and the resulting water falls out of the sample, the mass of the sample changes and heat is lost with the fallout of the water. This is known as a pseudo-adiabatic process. If all the condensed water remains in the

sample, then the process is of course reversible. In the atmosphere conditions are usually such that some, but not all, of the condensed water falls out of any sample of moist air. Using Eqs 2.1, 2.9 and 2.16, it may be shown that for saturated air which is lifted slightly,

$$\frac{\partial T}{\partial Z} = \gamma_s = \frac{\gamma_d \left(1 + \frac{L\, x_s}{R_d T}\right)}{1 + \frac{L^2 x_s}{R' C_p T^2}} \qquad (2.17)$$

where γ_s is the *saturated adiabatic lapse rate* (SALR) and x_s is the *saturated humidity mixing ratio*, that is, the mass of water vapor present in the moist air measured per gram of dry air when the moist air is saturated. Although γ_s varies with temperature and pressure, a typical value in the atmosphere is –5.0 °C km^{-1}.

2.7 Stability and convection in the atmosphere

Moist air can become saturated, and hence produce precipitation, by movement upwards in the atmosphere. Consider a parcel of moist air, for which the pressure in the parcel is the same as that of its environment. Assuming that the parcel can move vertically without disturbing the environment and does not mix with its environment, it can be shown from the equations of motion for the atmosphere that

$$\frac{dw}{dt} = -\frac{\partial p}{\partial Z} - g = \frac{(T - T')g}{T'} \qquad (2.18)$$

where w is the vertical velocity of the parcel, t is time, T is the temperature of the parcel, and T' is the temperature of the environment. Making the further assumption that the movement is adiabatic,

$$\frac{dw}{dt} = \frac{(T_0 - T_0')g}{T_0'} + \frac{(\gamma - \gamma_a)g\, z}{T_0'} \qquad (2.19)$$

where T_0 and T_0' are the initial temperature of the parcel and the environment respectively, γ is the environmental lapse rate, z is the vertical coordinate, and γ_a is the appropriate adiabatic lapse rate, being γ_d if the parcel is unsaturated and γ_s if the parcel is saturated. If the temperature is constant in the horizontal then $T_0 = T_0'$ and

$$\frac{dw}{dt} = \frac{(\gamma - \gamma_a)g\, z}{T_0'} \qquad (2.20)$$

The atmosphere is regarded as stable, neutral or unstable when *dw/dt* is <0, 0 or >0, respectively. If $\gamma_s < \gamma < \gamma_a$ then the atmosphere is *conditionally unstable*, whereas if $\gamma < \gamma_s$ the atmosphere is *absolutely stable* and if $\gamma > \gamma_d$ the atmosphere is *absolutely unstable*. Hence if moist air is lifted by some means it may become saturated and hence unstable, and may then continue to rise without any external force being applied.

2.8 The growth of precipitation particles

The condensation of water vapor in the atmosphere, brought about by the movement of air upwards, provides water droplets or ice crystals in clouds. Such precipitation particles are denser than the air surrounding them, and therefore they begin to fall at a rate of a few centimeters per second. However, these particles will either evaporate in unsaturated air below the cloud, or be held suspended by vertical currents within the

cloud. They will only be able to reach the ground as precipitation if they become large enough to stand evaporative losses and overcome upward air motions.

The droplet concentration is determined by the balance between the rate of supply of vapor by cooling and the rate at which it is removed by deposition. Increases in the supply of vapor, for example with increased vertical air velocity in more vigorous clouds, will result in an increase in the number and density of particles which are activated to become droplets. The concentration of droplets will be reduced by the entrainment of dry air from the environment.

The droplet concentration is crucial to the rate of growth of the droplets since they compete for the available water. Droplets grow more rapidly by condensation in clouds formed in relatively unpolluted regions than in clouds with high droplet concentrations.

In 1911 Wegener proposed that rain was formed by the melting of ice particles. The atmosphere normally contains many particles which can act as nuclei for the formation of water droplets. However, there are relatively few particles in the atmosphere which can act as nuclei for the formation of ice crystals except at low temperatures. Consequently, clouds rarely become glaciated, i.e. composed mainly of ice crystals rather than water droplets, until their temperature is much less than 0 °C, typically around –15 °C. At low temperatures, around –40 °C, nucleation of cloud droplets to form ice crystals will occur even without the presence of a nucleating particle, and therefore at such low temperatures very few clouds contain supercooled drops.

The relatively high saturation mixing ratio over water compared with that over ice, combined with lower concentrations of ice crystals than of water droplets, lead to much more rapid growth of ice crystals by deposition than is the case for droplets. When a cloud of water droplets in air at close to water saturation is cooled and a small fraction of the droplets freeze, these ice particles grow very rapidly as shown in Figure 2.2. The rapid growth of the ice crystals at close to water saturation reduces the vapor mixing ratio to a value between those of saturation with respect to ice and water. The result is evaporation of the water droplets.

Figure 2.2 Illustrating the growth of ice spheres and decay of water drops in a mixture subject to a constant updraught of 1 m s^{-1}. The air is assumed to be initially saturated with respect to water. The dashed line shows the growth of the droplets in the absence of any ice particles
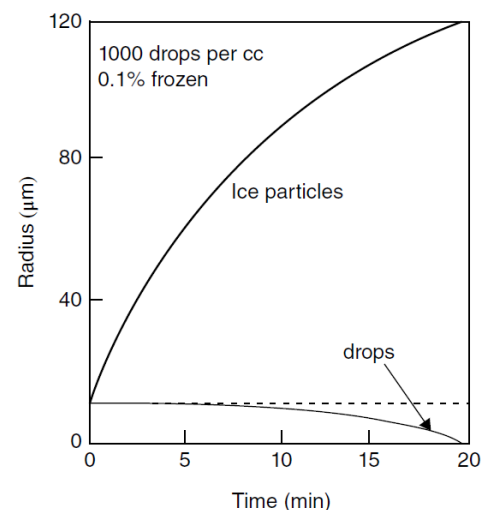
Ice crystals grow by sublimation when they exist in cloud together with supercooled water droplets. As water vapor is removed from the air by this process, the air becomes unsaturated with respect to water, so the droplets evaporate. This continues until either all the droplets have been evaporated or the ice crystals become so large that they fall from the cloud. This process takes from 10 to 30

minutes, and as the ice crystals fall they may melt to form rain (droplets with radii ≥20µm) which can reach the ground. A theory of this process was derived by Bergeron in 1935

This explains the formation of precipitation in mid-latitudes where clouds usually extend well above the 0 °C level; because the cloud particles normally begin as cloud condensation nuclei, it is referred to as the *cold rain process*. However, it is observed that warm clouds with tops below 0 °C also produce rain. Indeed, such clouds occur in mid-latitudes as well as in the tropics. A *warm rain process* is required. It was discovered that cloud particles normally begin to form on *cloud condensation nuclei* (CCN), which consist of partially or completely soluble aerosol particles. As the cloud particles grow by condensation or sublimation on the CCN, they begin to fall and collect other particles. The type of precipitation which is formed by such collisions depends upon the types of cloud particles present. If the cloud contains only water then rain is formed and the process is known as *coalescence*. However, if only ice crystals exist then snow results and the process is called *aggregation*.

Owing to the very slow rate of growth of droplets by condensation once they have reached about 20 µm in radius, subsequent growth is mainly by the collision and aggregation process. The collection efficiencies of smaller cloud droplets are relatively low, as is the rate at which droplets encounter one another, owing to the small values of the terminal velocities.

If both water droplets and ice crystals exist then ice and snow pellets or hail may form and the process is accretion. Eventually the precipitation particles may reach such a size that they break up, beginning the process again. They are sometimes collectively referred to as the *Langmuir process* in recognition of the work of Langmuir in 1948. This process may exist together with the Bergeron process. A summary is given in Figure 2.3.

Figure 2.3 The evolution of warm and cold rain starting from cloud condensation nuclei (CCN) and ice nuclei (IN)