A.2 Homework 2 + Solutions

Homework 2

Q1 Find the abstract group structure of $\mathcal{U}(\mathbb{F}_2 D_{12})$. Hints :

1 Note that Maschke's theorem does not apply.

2 $D_{12} \cong C_2 \times D_6$.

3 $\mathcal{U}(\mathbb{F}_2 D_6) \cong D_{12}$

Q2 Find the size of the group $\mathcal{U}(\mathbb{F}_2 D_{12})$. Hint : $|\mathcal{U}(\mathbb{F}_3 D_6)| = 324$.

Q3 (a) Show that $D_8^{'} \cong C_2$.

(b) Show that $D_8/D_8^{'} \cong C_2 \times C_2$.

(c) Conclude that $\mathbb{F}_p D_8 \cong (\oplus_{i=1}^{4} \mathbb{F}_p) \oplus M_2(\mathbb{F}_p)$. (where $p \neq 2$).

Q4 (a) Find all the conjugacy classes of D_8 (there are 5).

(b) What is $\dim_{\mathbb{F}_p} Z(\mathbb{F}_p D_8)$.

(c) Conclude that $\mathbb{F}_p D_8 \cong (\oplus_{i=1}^{4} \mathbb{F}_p) \oplus M_2(\mathbb{F}_p)$. (where $p \neq 2$).

Q5 Let R be a commutative ring and let G and H be groups. Prove that

$$R(G \times H) \cong (RG)H.$$

Q6 Let $\{R_i\}_{i \in I}$ be a set of rings and let G be a group. Let $R = \oplus_{i \in I}$. Show that $RG \cong \oplus_{i \in I} R_i G$.

Q7 The quaternion group of 8 elements has the following presentation:

$$\mathbb{H} = < a, b \,|\, a^4 = 1,\ a^2 = b^2,\ bab^{-1} = a^{-1} >$$

(a) Show that $\mathbb{H}^{'} = < a^2 >$

(b) Show that $\mathbb{H}/\mathbb{H}^{'} \cong C_2 \times C_2$.