Theorem 4.44 *Let $\{R_i\}_{i\in I}$ be a set of rings and let $R = \oplus_{i\in I} R_i$. Let G be a group. Then*

$$RG \cong (\oplus_{i\in I} R_i)G \cong \oplus_{i\in I}(R_i G).$$

Proof. Homework 2. ∎

Example 4.45 $\mathbb{F}_5 C_6$. $\mathbb{F}_5 C_6 \cong \mathbb{F}_5(C_2 \times C_3) \cong (\mathbb{F}_5 C_2)C_3 \cong (\mathbb{F}_5 \oplus \mathbb{F}_5)C_3 \cong \mathbb{F}_5 C_3 \oplus \mathbb{F}_5 C_3$.

Now $\mathbb{F}_5 C_3 \cong \mathbb{F}_5 \oplus \mathbb{F}_5 \oplus \mathbb{F}_5$ *or* $\mathbb{F}_5 C_3 \cong \mathbb{F}_5 \oplus \mathbb{F}_{5^2}$. $\therefore \mathcal{U}(\mathbb{F}_5 C_3) \cong C_4 \times C_4 \times C_4$ *or* $C_4 \times C_{24}$. *But* $C_3 < \mathcal{U}(\mathbb{F}_5 C_3)$, *so by lagrange's theorem* , $3 \mid \mathcal{U}(\mathbb{F}_5 C_3)$. *However* $3 \nmid |C_4 \times C_4 \times C_4|$ *and* $3 \mid |C_4 \times C_{24}|$ *so* $\mathcal{U}(\mathbb{F}_5 C_3) \cong C_4 \times C_{24}$ *and* $\mathbb{F}_5 C_3 \cong \mathbb{F}_5 \oplus \mathbb{F}_{5^2}$.

$$\begin{aligned} \therefore \mathbb{F}_5 C_6 &\cong \mathcal{U}(\mathbb{F}_5 C_3) \oplus \mathcal{U}(\mathbb{F}_5 C_3) \\ &\cong \mathbb{F}_5 \oplus \mathbb{F}_{5^2} \oplus \mathbb{F}_5 \oplus \mathbb{F}_{5^2} \\ &\cong \mathbb{F}_5 \oplus \mathbb{F}_5 \oplus \mathbb{F}_{5^2} \oplus \mathbb{F}_{5^2} \end{aligned}$$

Theorem 4.46 (***Fundamental Theorem of Finite Abelian Groups***) *Let A be a finite abelian group. Then*

$$A \cong G_1 \times G_2 \times \cdots \times G_n$$

, where G_i is a cyclic group of order ${p_i}^{m_i}$, where p_i is some prime.

Example 4.47 *Let A be an abelian group of order $30 = 2^1.3^1.5^1$. Then*

$$\begin{aligned} A &\cong C_{30} \\ &\cong C_5 \times C_6 \\ &\cong C_5 \times C_3 \times C_2 \\ &\cong C_{15} \times C_2 \\ &\cong C_{10} \times C_3 \end{aligned}$$

These are all the same because 2,3 and 5 are all relatively prime.

$$\therefore A \cong C_2 \times C_3 \times C_5.$$