$$\begin{aligned}\therefore \mathbb{F}_5 S_3 &\cong \mathbb{F}_5 C_2 \oplus NCP \\ &\cong \mathbb{F}_5 \oplus \mathbb{F}_5 \oplus NCP \\ &\cong \mathbb{F}_5 \oplus \mathbb{F}_5 \oplus M_2(\mathbb{F}_5).\end{aligned}$$

$$\begin{aligned}\therefore Z(\mathbb{F}_5 S_3) &\cong Z(\mathbb{F}_5 \oplus \mathbb{F}_5 \oplus M_2(\mathbb{F}_5)) \\ &\cong \mathbb{F}_5 \oplus \mathbb{F}_5 \oplus Z(M_2(\mathbb{F}_5)) \\ &\cong \mathbb{F}_5 \oplus \mathbb{F}_5 \oplus I_{2\times 2}.\mathbb{F}_5 \\ &\cong \mathbb{F}_5 \oplus \mathbb{F}_5 \oplus \mathbb{F}_5.\end{aligned}$$

This is a 3-dimensional vector space over $\mathbb{F}_5$ (with basis $\{(1,0,0),\ (0,1,0),\ (0,0,1)\}$). $\therefore$ S_3 has 3 conjugacy classes. We proved this group theory result using group rings.

Now using group theory, find the 3 conjugacy classes of S_3.

Theorem 4.41 *Let R be a commutative ring and let G and H be groups. Then*

$$R(G \times H) \cong (RG)H.$$

Proof. Homework 2. ■

Corollary 4.42

$$R(G \times H) \cong (RG)H \cong (RH)G$$

Proof. $R(G\times H) \cong R(H\times G)$ and now use the theorem. Note $G\times H \cong H\times G$ by $(g,h) \mapsto (h,g)$. ■

Corollary 4.43

$$R(G_1 \times G_2 \times \cdots \times G_n) \cong (((RG_1)G_2)\ldots)G_n$$