$\therefore Z(RG) = \{$linear combinations of $\widehat{C}_i$ over $R\}$.

It remains to show linear independance of $\{\widehat{C}_i\}$. Suppose $\sum_{i\in I} c_i\widehat{C}_i = 0$. Then we have an R-linear combination of elements of G, but the elements of G are linear independant over R. So the coefficients are all 0.

$$\sum_{i\in I} c_i\widehat{C}_i = 0 \Longrightarrow c_i = 0 \ \forall\, i \in I$$

$\therefore \{\widehat{C}_i\}$ is linear independant over R. ■

Recall the class equation of a finite group G. Let $\{x_1, x_2, \ldots, x_t\}$ be a complete set of conjugacy class representatives of G. Let $c(x_i)$ = conjugacy class containing x_i. Let $n_i = |C(x_i)| = [G : C_G(x_i)]$. Then $|G| = \sum_{i=1}^{t} n_i$ $= \sum_{i=1}^{t} |C(x_i)| = \sum_{i=1}^{t} [G : C_G(x_i)] = |Z(G)| + \sum_{n_i>1} n_i$. (Note : $n_i = 1 \Longleftrightarrow x_i \in Z(G)$).

Lemma 4.38 *Let G be a finite group and $\mathbb{C}$ the complex numbers. Then*

$$\mathbb{C}G \cong \oplus_{i=1}^{t} M_{n_i}(\mathbb{C})$$

where $t =$ the number of conjugacy classes of G.

Proof. $\dim_{\mathbb{C}} \mathbb{C}G = \sharp$ of conjugacy classes of G. $\therefore \dim_{\mathbb{C}} Z(\oplus_{i=1}^{t} M_{n_i}(\mathbb{C}))$ $= \sum_{i=1}^{t} \dim_{\mathbb{C}} Z(M_{n_i}(\mathbb{C})) = \sum_{i=1}^{t} 1 = t$. ■

Example 4.39 $\mathbb{F}_5C_2 \cong \mathbb{F}_5 \oplus \mathbb{F}_5$. *Here $Z(\mathbb{F}_5C_2) = \mathbb{F}_5C_2$ so $dim_{\mathbb{F}_5} Z(\mathbb{F}_5C_2) = dim_{\mathbb{F}_5}(\mathbb{F}_5C_2) = 2 = \sharp$ of conjugacy classes of C_2. ($C_2 = \{1, x\} \Longrightarrow \{1\}$ and $\{x\}$ are the only conjugacy classes of C_2).*

Example 4.40 $\mathbb{F}_5S_3 \cong \mathbb{F}_5 \oplus \mathbb{F}_5 \oplus M_2(\mathbb{F}_5)$. $S_3 =< x, y \,|\, x^n = y^2 = 1,\ yxy = x^{-1} >$. $S_3^{'} =< x^2 >\cong C_3$. $\therefore S_3\ S_3^{'} \cong C_2$