$$\mathbb{F}_5 D_{12} \cong \mathbb{F}_5(C_2 \times C_2) \oplus NCP$$
$$\mathbb{F}_5 D_{12} \cong \mathbb{F}_5 \oplus \mathbb{F}_5 \oplus \mathbb{F}_5 \oplus \mathbb{F}_5 \oplus NCP$$

$\therefore$ NCP has dimension 8. So $NCP \cong M_2(\mathbb{F}_5) \oplus M_2(\mathbb{F}_5)$ or $NCP \cong M_2(\mathbb{F}_{5^2})$.

So $\mathcal{U}(\mathbb{F}_5 D_{12}) \cong C_4 \times C_4 \times C_4 \times C_4 \times GL_2(\mathbb{F}_5) \times GL_2(\mathbb{F}_5)$ or $\mathcal{U}(\mathbb{F}_5 D_{12}) \cong C_4 \times C_4 \times C_4 \times C_4 \times GL_2(\mathbb{F}_{5^2})$.

$$|\mathcal{U}(\mathbb{F}_5 D_{12})| = (p-1)^4\{(p^2-1)(p^2-p)\}^2 = 4^4\{(24)(20)\}^2 = 2^{18}3^2 5^2$$

or

$$|\mathcal{U}(\mathbb{F}_5 D_{12})| = (p-1)^4\{(q^2-1)(q^2-q)\} = 4^4\{((5^2)^2-1)((5^2)^2-5^2)\}$$

Note that $D_{12} < \mathcal{U}(\mathbb{F}_5 D_{12}$ so $12 \mid |\mathcal{U}(\mathbb{F}_5 D_{12})|$. But 12 divides the order of both cases so this does not help to differentiate between them. Also, $U = \mathcal{U}(\mathbb{F}_5 D_{12}) \cong \mathcal{U}(\mathbb{F}_5(D_6 \times C_2)) > \mathcal{U}(\mathbb{F}_5 D_6)$ and $U > \mathcal{U}(\mathbb{F}_5 C_2)$.

Lemma 4.35 *$Z(M_n(K)) = I_{n\times n}.K$. Thus $dim_K(Z(M_n(K))) = 1$.*

Definition 4.36 *Let G be a finite group and R a commutative ring. Let $\{C_i\}_{i\in I}$ be the set of conjugacy classes of G. Then*

$$\widehat{C_i} = \sum_{c\in C_i} c \in RG$$

*is called the **class sum** of C_i.*

Theorem 4.37 *Let G be a group and R a commutative ring. Then the set of class sums $\{\widehat{C_i}\}$ of G forms a basis for $Z(RG)$ over R. Thus $Z(RG)$ has dimension t over R, where t is the number of conjugacy classes of G.*

Proof. Let $\widehat{C_i}$ be a class sum. Let $g \in G$. Then $\widehat{C_i}^g = \widehat{C_i}$. $\therefore \widehat{C_i} \in Z(RG)$. Let $\alpha = \sum a_g g \in Z(RG)$. Let $h \in G$. Then $\alpha^h = \alpha$ so $a_{g^h} = a_g$ (coefficient of g = coefficient of g^h). Thus the entire conjugacy class C_i has the same coefficient in the expansion of α. $\therefore \alpha = \sum_{i\in I} c_i\widehat{C_i}$ $(c_i \in R)$. $\therefore Z(RG) \subset \{$linear combinations of $\widehat{C_i}$ over $R\}$.