$\mathbb{F}_5 \oplus_{i=1}^{s-1} M_{n_i}(K_i)$. $D_{12} =< x, y \,|\, x^6 = y^2 = 1,\ yxy = x^5 >$. ${D_{12}}' = ?$

$$
\begin{aligned}
[x^i y^j, x^k y^l] &= y^{-j}x^{-i}y^{-l}x^{-k}x^i y^j x^k y^l \quad i,k \in \{0,1,2,3,4,5\}\ j,l \in \{0,1\} \\
&= y^j x^{-i} y^l x^{-k} x^i y^j x^k y^l \\
&= x^{(-i)(-1)j} y^{j+l} x^{i-k} y^j x^k y^l \\
&= x^{(-i)j(-1)} x^{(i-k)(-1)(j+l)} y^{j+j+l} x^k y^l \\
&= x^{(-i)j(-1)+(i-k)(-1)(j+l)} x^{k(-1)(2j+l)} y^{2j+2l} \\
&= x^{(-i)j(-1)+(i-k)(-1)(j+l)+k(-1)(2j+l)}.1 \\
&= x^{[(-i)j(-1)+(i)(-1)(j+l)]+[(-k)(-1)(j+l)+k(-1)(2j+l)]} \\
&= x^{i\{(-1)j(-1)+(-1)(j+l)\}+k\{(-1)(-1)(j+l)+(-1)(2j+l)\}}
\end{aligned}
$$

Now consider a number of cases

(*i*) *j and l even :*

$$[\ ,\ ] = x^{i\{-1+1\}+k\{(-1)+1\}} = x^0 = 1$$

(*ii*) *j even and l odd :*

$$[\ ,\ ] = x^{i\{-1+(-1)\}+k\{1+(-1)\}} = x^{-2i}$$

(*iii*) *j odd and l even :*

$$[\ ,\ ] = x^{i\{1+(-1)\}+k\{1+1\}} = x^{2k}$$

(*iii*) *j and l odd :*

$$[\ ,\ ] = x^{i\{1+1\}+k\{-1+(-1)\}} = x^{2i-2k}$$

$$\therefore {D_{12}}' = \{1, x^2, x^4\} \cong C_3$$

$$\therefore D_{12}/{D_{12}}' \cong C_4 \text{ or } C_2 \times C_2 \text{ (considering sizes)}$$

Note : *$D_{12} \cong D_6 \times C_2$ also $C_{12} \ncong C_6 \times C_2$ but $C_{12} \cong C_3 \times C_4$.* $D_{12} \cong D_6 \times C_2 =< x^2, y \,|\, (x^2)^3 = y^2 = 1,\ y(x^2)y = (x^2)^{-1} > \times < x^3 > = \{x^{2i}.y^j.x^{3k} \,|\, i \in \{0,1,2\},\ j \in \{0,1\},\ k \in \{0,1\}\}$.

$$\therefore \frac{D_{12}}{{D_{12}}'} \cong \frac{D_6 \times C_2}{C_3} \cong \frac{D_6}{C_3} \times C_2 = C_2 \times C_2$$