Proof. **(i)** $H < G$.

$$\begin{aligned} {e_H}^2 &= \frac{1}{|H|}.\widehat{H}\frac{1}{|H|}.\widehat{H} \\ &= \frac{1}{|H|^2}\sum_{i=1}^{n} h_i\widehat{H} \quad \text{where } |H| = n. \\ &= \frac{1}{|H|^2}\sum_{i=1}^{n}\widehat{H} \\ &= \frac{1}{|H|^2}.n.\widehat{H} \\ &= \frac{1}{|H|^2}.|H|.\widehat{H} \\ &= \frac{1}{|H|}.\widehat{H} = e_H \end{aligned}$$

(ii) Let $H \triangleleft G$. We will show that e_H commutes with every element of RG. It suffices to show that e_H commutes with every element of G. So we must show that ${e_H}^g = g^{-1}e_H g = e_H \ \forall\ g \in G$. Now ${e_H}^g = g^{-1}\frac{1}{|H|}.\widehat{H}g$ $= \frac{1}{|H|}g^{-1}(h_1 + h_2 + \cdots + h_n)g = \frac{1}{|H|}(h_1 + h_2 + \cdots + h_n) = e_H$. ∎

Definition 4.24 *Let X be a subset of RG. Then the* ***left-annihilator*** *of X in RG is*

$$ann_l(X) = \{\alpha \in RG \,|\, \alpha.x = 0 \ \forall\ x \in X\}$$

Similarly we can define the ***right-annihilator*** *of X in RG is*

$$ann_r(X) = \{\alpha \in RG \,|\, x.\alpha = 0 \ \forall\ x \in X\}$$

Definition 4.25 $\Delta_R(G, H) = \{\sum_{h\in H} \alpha_h(h-1) \,|\, \alpha_h \in RG\}$ *We usually write* $\Delta_R(G, H) = \Delta(G, H)$.

Note : $\Delta(G, H) \overset{l}{\triangleleft} RG$ (left ideal, check).
Note : $\Delta(G, G) = \Delta(G)$.