$\therefore \oplus_{i=1}^{s-1} M_{n_i}(K_i)$ *is a 5-dimensional vectors space over* $\mathbb{F}_5$. *But* $\mathbb{F}_5 S_3$ *is non-commutative so* $n_i > 1 \ \exists \ i$.

$$\therefore \oplus_{i=1}^{s-1} M_{n_i}(K_i) = \mathbb{F}_5 \oplus M_2(\mathbb{F}_5)$$

$$\therefore \mathbb{F}_5 S_3 \cong \oplus_{i=1}^{s} M n_i(K_i) \cong \mathbb{F}_5 \oplus \mathbb{F}_5 \oplus M_2(\mathbb{F}_5)$$

$$\therefore \mathcal{U}(\mathbb{F}_5 S_3) \cong \mathcal{U}(\mathbb{F}_5) \times \mathcal{U}(\mathbb{F}_5) \times \mathcal{U}(M_2(\mathbb{F}_5)) \cong C_4 \times C_4 \times GL_2(\mathbb{F}_5)$$

$GL_2(\mathbb{F}_5) = \{A \in M_2(\mathbb{F}_5) \mid det\ A = 0\} = \{A \in M_2(\mathbb{F}_5) \mid$ *rows of A are linearly independant.*
Check : $\begin{pmatrix} a & b \\ c & d \end{pmatrix}^{-1} = \dfrac{1}{ad-bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$. *Now let's count the size of* $GL_2(\mathbb{F}_5)$:

There are $5^2 - 1 = 24$ *choices for the first row (not including the zero row) and there are* $5^2 - 5 = 20$ *choices for the second row (not a multiple of the first row).* $\therefore |GL_2(\mathbb{F}_5)| = (5^2-1)(5^2-5) = 480$. $\therefore \mathcal{U}(\mathbb{F}_5 S_3)$ *has order* $4.4.480 = 7680$.

Theorem 4.20 $GL_2(\mathbb{F}_p)$ *is a non abelian group of order* $(p^2-1)(p^2-p)$. $GL_2(\mathbb{F}_{p^n})$ *is a non abelian group of order* $(p^{2n}-1)(p^{2n}-p^n)$. $GL_3(\mathbb{F}_{p^n})$ *is a non abelian group of order ? (Homework).*

Definition 4.21 *Let* $x \in G$ *be an element of order* n *(i.e.* $x^n = 1$*). Then define*

$$\widehat{x} = 1 + x + x^2 + \cdots + x^{n-1} \in RG$$

Definition 4.22 *Let* $H < G$ *(H-finite so* $H = \{h_1, h_2, \ldots, h_n\}$*). Then define*

$$\widehat{H} = h_1 + h_2 + \cdots + h_n \in RH \subset RG.$$

So $\widehat{x} = \langle x \rangle \in R\langle x \rangle \subset RG$.

Lemma 4.23 *Let H be a finite subgroup of G and R any ring (with unity). If* $|H|$ *is invertible in R then* $e_H = \dfrac{1}{|H|}.\widehat{H} \in RH$ *is an idempotent. Moreover if* $H \lhd G$ *then* $e_H = \dfrac{1}{|H|}.\widehat{H}$ *is central in RG.*