Counting dimensions we see that $3 = \sum_{i=1}^{s} {n_i}^2 = \sum_{i=1}^{3} 1^2$. $\therefore D_i = \mathbb{C}$, $n_i = 1 \; \forall \; i$ *and* $s = 3$. $\therefore \mathbb{C}C_3 \cong \mathbb{C} \oplus \mathbb{C} \oplus \mathbb{C}$. $\therefore \mathcal{U}(\mathbb{C}C_3) \cong \mathcal{U}(\mathbb{C} \oplus \mathbb{C} \oplus \mathbb{C}) = \mathcal{U}(\mathbb{C}) \times \mathcal{U}(\mathbb{C}) \times \mathcal{U}(\mathbb{C})$.

The zero divisors of $\mathbb{C}C_3$ *correspond bijectively to the zero divisors of* $\mathbb{C} \oplus \mathbb{C} \oplus \mathbb{C}$

$$= \{(a,b,0) \,|\, a,b \in \mathbb{C}\} \cup \{(a,0,c) \,|\, a,c \in \mathbb{C}\} \cup \{(0,b,c) \,|\, b,c \in \mathbb{C}\}$$

Example 4.11 $\mathbb{C}S_3$. *S_3 is finite and $\mathbb{C} = 0 \nmid 6$ so maschke's theorem does apply and*

$$\mathbb{C}S_3 \cong \oplus_{i=1}^{s} M_{n_i}(D_i) = \oplus_{i=1}^{s} M_{n_i}(\mathbb{C})$$

$6 = 1^2 + 1^2 + 2^2$ *or* $6 = \sum_{i=1}^{6} 1^2$. *So* $\mathbb{C}S_3 \cong \mathbb{C} \oplus \mathbb{C} \oplus M_2(\mathbb{C})$ *or* $\mathbb{C}S_3 \cong \mathbb{C} \oplus \mathbb{C} \oplus \mathbb{C} \oplus \mathbb{C} \oplus \mathbb{C} \oplus \mathbb{C}$. *But* $\oplus_{i=1}^{6}\mathbb{C}$ *is a commutative ring so* $\mathbb{C}S_3 \not\cong \oplus_{i=1}^{6}\mathbb{C}$.

$\therefore \mathbb{C}S_3 \cong \mathbb{C} \oplus \mathbb{C} \oplus M_2(\mathbb{C})$ *and* $\therefore \mathcal{U}(\mathbb{C}S_3) \cong \mathcal{U}(\mathbb{C}) \times \mathcal{U}(\mathbb{C}) \times GL_2(\mathbb{C})$. *The zero divisors of* $\mathbb{C}S_3$ *correspond bijectively to the zero divisors of* $\mathbb{C} \oplus \mathbb{C} \oplus M_2(\mathbb{C})$.

$$\begin{aligned} &= \{(a,b,A) \,|\, a,b, \in \mathbb{C},\ A \in \mathcal{ZD}(M_2(\mathbb{C}))\} \\ &= \{(a,0,A) \,|\, a, \in \mathbb{C},\ A \in \mathcal{ZD}(M_2(\mathbb{C}))\} \cup \{(0,b,A) \,|\, b, \in \mathbb{C},\ A \in \mathcal{ZD}(M_2(\mathbb{C}))\} \end{aligned}$$

Example 4.12 $\mathbb{F}_2C_2$ *does not compose as* $\oplus_{i=1}^{s} M_{n_i}(D_i)$ *since* $2|2$ *(i.e char* $\mathbb{F}_2 \,|\, |G|$*).*

Theorem 4.13 (***Wedderburn***) *A finite division ring is a field.*

Example 4.14 $\mathbb{F}_3C_2$. *Maschke's theorem applies since* $|C_2| < \infty$ *and char* $\mathbb{F}_3 \nmid |C_2|$. $\therefore \mathbb{F}_3C_2 \cong \oplus_{i=1}^{s} M_{n_i}(D_i)$. $2 = \sum_{i=1}^{s}({n_i}^2.\ dim_{\,\mathbb{F}_3}(D_i))$. *Note that* $\mathbb{F}_3$ *is not algebraically closed (check). So we need* $dim_{\,\mathbb{F}_3}(D_i)$. *Now* $2 = 1 + 1 = 1.2$. *So* $dim_{\,\mathbb{F}_3}(D) = 1$ *or* 2. $\therefore \mathbb{F}_3C_2 \cong \mathbb{F}_3 \oplus \mathbb{F}_3$ *or* $\therefore \mathbb{F}_3C_2 \cong D$ *where* $dim_{\,\mathbb{F}_3}(D) = 2$.

$$\therefore \mathbb{F}_3C_2 \cong \mathbb{F}_3 \oplus \mathbb{F}_3 \text{ or } \mathbb{F}_{3^2}$$

Question : Which one is it ?