(*i*) *R is semisimple*

(*ii*) *G is finite*

(*iii*) *$|G|$ is invertible in R.*

Corollary 4.5 *Let G be a group and K a field. Then KG is semisimple if and only if G is finite and the characteristic $K \nmid |G|$.*

Proof. First note that any field K is semisimple ($K = M_1(K)$ and use a previous lemma).
($\Leftarrow$) Let $|G| < \infty$ and char$K \nmid |G|$. So $|G| \in K \setminus \{0\}$.
($\Rightarrow$) $|G|$ is invertible in K. Now apply maschke's theorem $\Longrightarrow$ let KG be semisimple. G is finite by maschke's and also $|G|$ is invertible in K so $|G| \in K \setminus \{0\}$. So $|G|$ is not a multiple of char $K \in K$. $\therefore K \nmid |G|$. ■

Theorem 4.6 *Let G be a finite group and K a finite field such that char $K \nmid |G|$. Then $KG \cong \oplus_{i=1}^{s} M_{n_i}(D_i)$ where D_i is a division ring containing K in it's center and*

$$|G| = \sum_{i=1}^{s}(n_i{}^2.dim_K(D_i))$$

Definition 4.7 *A field K is* **algebraically closed** *if it contains all of the roots of the polynomials in $K[x]$.*

Example 4.8 $\mathbb{C}$ *is algebraically closed, while* $\mathbb{H}$ *is not.*

Corollary 4.9 *Let G be a finite group and K an algebraically closed field, where char $K \nmid |G|$. Then*

$$KG \cong \oplus_{i=1}^{s} M_{n_i}(K) \quad and \quad |G| = \sum_{i=1}^{s} n_i{}^2$$

Example 4.10 $\mathbb{C}C_3$. *Note that C_3 is finite and char $\mathbb{C} = 0 \nmid 3$ so maschke's theorem does apply and*

$$\mathbb{C}C_3 \cong \oplus_{i=1}^{s} M_{n_i}(D_i) = \oplus_{i=1}^{s} M_{n_i}(\mathbb{C}) \text{ by the corollary above}$$