$= \sum_{i=1}^{n} \eta_i$ where η_i is an n^{th} roots of unity.

$$\begin{aligned}
\therefore nc_1 &= \sum_{i=1}^{n} \eta_i \\
\therefore |nc_1| &= |\sum_{i=1}^{n} \eta_i| \leq \sum_{i=1}^{n} |\eta_i| = n. \\
\therefore |c_1| &\leq 1 \Longrightarrow c_1 = \pm 1 \\
\therefore nc_1 &= \sum_{i=1}^{n} \eta_i = n \text{ or } -n, \text{ so } \eta_i = \eta_i \ \forall\, i \\
\text{so } nc_1 &= n\eta_i \Longrightarrow \eta_i = \pm 1 \ \forall\, i \\
\therefore \mathcal{T}(\gamma) &\backsim D = I \text{ or } I \\
\therefore \mathcal{T}(\gamma) &= I \text{ or } I
\end{aligned}$$

But $\mathcal{T} : \mathbb{C}G \longrightarrow M_n(\mathbb{C})$ is injective, so $\gamma = \pm 1$ $(= c_1)$. ■

Corollary 3.11 *Let* $\gamma \in Z(\mathcal{U}(\mathbb{Z}G))$ *where* $\gamma^m = 1$ *and* G *is finite. Then* $\gamma = \pm g \,\exists\, g \in G$. *(i.e. all central torsion units are trivial).*

Proof. Let $\gamma \in Z(\mathcal{U}(\mathbb{Z}G))$ with $\gamma^m = 1$ and $|G| = n$. Let $\gamma = \sum_{i=1}^{n} c_{g_i} g_i$ and let $c_{g_2} \neq 0 \,\exists\, g_2 \in G$. $\therefore \gamma {g_2}^{-1} = \sum_{i=1}^{n} c_{g_i} g_i {g_2}^{-1}$ $(\star)$ is a unit of finite order in $\mathbb{Z}G$ (Let ${g_2}^{m_2} = 1$, then $(\gamma {g_2}^{-1})^{m.m_2} = \gamma^{m.m_2}({g_2}^{-1})^{m.m_2} = 1.1 = 1$ since γ is central).

Now from $(\star)$ the coefficient of 1 in $\gamma {g_2}^{-1}$ is $c_{g_2} \neq 0$. Now applying the Berman-Higman theorem to $\gamma {g_2}^{-1}$ to get that

$$\gamma {g_2}^{-1} = \pm 1 = c_{g_2} \Longrightarrow \gamma = \pm 1.g_2 = \pm g_2 \ \exists\ g_2 \in G$$

■

Theorem 3.12 (***Higman***) *Let* A *be a finite abelian group. Then the group of torsion units of* $\mathbb{Z}A$ *equals* $\pm A$.