

$\therefore RG = \{\lambda_1.1 + \lambda_2.a + \lambda_3.a^2 \mid \lambda_i \in R\}$ . What does  $g.\alpha$  look like (where  $g \in G$  and  $\alpha \in RG$ ) ?

$$\begin{aligned} 1(\lambda_1.1 + \lambda_2.a + \lambda_3.a^2) &= \lambda_1.1 + \lambda_2.a + \lambda_3.a^2 \\ (*) a(\lambda_1.1 + \lambda_2.a + \lambda_3.a^2) &= \lambda_3.1 + \lambda_1.a + \lambda_2.a^2 \\ (**) a^2(\lambda_1.1 + \lambda_2.a + \lambda_3.a^2) &= \lambda_2.1 + \lambda_3.a + \lambda_1.a^2 \end{aligned}$$

Correspondance

$$1 \longleftrightarrow \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, a \longleftrightarrow \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, a^2 \longleftrightarrow \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

(these are the basis elements which are acted upon, permuted by left-multiplication by  $3 \times 3$  matrices).

$$\mathcal{T} : 1 \longrightarrow \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

$$a \longrightarrow \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \text{ from } (*) a(\lambda_1.1 + \lambda_2.a + \lambda_3.a^2) \longleftrightarrow a \begin{pmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{pmatrix} = \begin{pmatrix} \lambda_3 \\ \lambda_1 \\ \lambda_2 \end{pmatrix},$$

$$a^2 \longrightarrow \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} \text{ from } (**) a^2(\lambda_1.1 + \lambda_2.a + \lambda_3.a^2) \longleftrightarrow a^2 \begin{pmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{pmatrix} = \begin{pmatrix} \lambda_2 \\ \lambda_3 \\ \lambda_1 \end{pmatrix}.$$