Also

$$\frac{RG}{IG} \cong \left(\frac{R}{I}\right) G.$$

Proof. (a) IG is a commutative group under $+$ ✓. Let $\alpha = \sum_{g \in G} a_g g \in IG$ and $\beta = \sum_{h \in G} b_h h \in RG$ (so $a_g \in I$ and $b_h \in R$ forall $g, h \in G$).

$$\alpha\beta = \left(\sum_{g \in G} a_g g\right)\left(\sum_{h \in G} b_h h\right) = \sum_{g,h \in G} \underbrace{a_g b_h}_{\in I} gh \in IG$$

So IG is an ideal of RG.

(b) $\frac{RG}{IG} = \{\beta + IG \mid \beta \in RG\}$ and $\left(\frac{R}{I}\right) G = \{\sum_{g \in G} (a_g + I) g \mid a_g + I \in \frac{R}{I}\}$. i.e. $a_g \in R$ and $g \in G$. Define

$$\theta : \frac{RG}{IG} \longrightarrow \left(\frac{R}{I}\right) G$$

by $\theta(\beta + IG) = \theta\left(\sum_{g \in G} b_g g + IG\right) = \sum_{g \in G} (b_g + I) G$. We must show that θ is an isomorphism.

$\theta(\alpha + IG + \beta + IG) = \theta(\alpha + \beta + IG) = \theta(\sum (a_g + b_g + IG) = \sum (a_g + b_g + I) g$.
Also $\theta(\alpha + IG) + \theta(\beta + IG) = \sum (b_g + I) g + \sum (a_g + I) g = \sum (a_g + b_g + I) g$ ✓.
$\theta((\alpha + IG)(\beta + IG)) = \theta(\alpha\beta + IG) = \theta(\sum_{g \in G} a_g g \sum_{h \in G} b_h h + IG) = \sum_{g,h \in G} (a_g$ [illegible]
Also $\theta(\alpha + IG)\theta(\beta + IG) = (\sum (a_g + I) g)(\sum (b_h + I) h) = \sum (a_g + I)($[illegible] $I) gh = \sum (a_g b_h + I) gh$ ✓. $\therefore \theta$ is a ring homomorphism. It remains to show that θ is bijective but we will do this on homework 2. ∎