$$
\begin{aligned}
\mathbb{F}_2C_2 &= \{\sum_{g\in C_2} a_g g \,|\, a_g \in \mathbb{F}_2\} \\
&= \{0_{\mathbb{F}_2}.1_{C_2} + 0_{\mathbb{F}_2}.x, 1_{\mathbb{F}_2}.1_{C_2} + 0_{\mathbb{F}_2}.x, 0_{\mathbb{F}_2}.1_{C_2} + 1_{\mathbb{F}_2}.x, 1_{\mathbb{F}_2}.1_{C_2} + 1_{\mathbb{F}_2}.x\} \\
&= \{0_{\mathbb{F}_2C_2}, 1_{\mathbb{F}_2C_2}, 1_{\mathbb{F}_2}.x, 1_{\mathbb{F}_2}.1_{C_2} + 1_{\mathbb{F}_2}.x\} \\
&= \{0, 1, x, 1+x\}
\end{aligned}
$$

Note that . is $\mathbb{F}_2$ module multiplication. Now let's construct the cayley tables for $\mathbb{F}_2C_2$.

$\underline{\underline{\mathbb{F}_2C_2}}$

$+$	0	1	x	$1+x$
0	0	1	x	$1+x$
1	1	0 $(\bullet)$	$1+x$	x
x	x	$1+x$	0 $(\star)$	1
$1+x$	$1+x$	x	1	0

$(\bullet)$ $1 + 1 = 1_{\mathbb{F}_2}.1_{C_2} + 1_{\mathbb{F}_2}.1_{C_2}$

$= (1_{\mathbb{F}_2} + 1_{\mathbb{F}_2})1_{C_2}$

$= (0_{\mathbb{F}_2})1_{C_2} = 0$

$(\star)$ $x + x = 1_{\mathbb{F}_2}.x + 1_{\mathbb{F}_2}.x$

$= (1_{\mathbb{F}_2} + 1_{\mathbb{F}_2})x$

$= (0_{\mathbb{F}_2})x = 0$

$(\mathbb{F}_2C_2, +)$ is a group.

$\underline{\underline{\mathbb{F}_2C_2}}$

$\cdot$	0	1	x	$1+x$
0	0	0	0	0
1	0	1	x	$1+x$
x	0	x	1	$1+x$
$1+x$	0	$1+x$	$1+x$	0 $(\bullet)$

$(\bullet)$ $(1 + x)(1 + x) = 1(1 + x) + x(1 + x)$

$= 1 + x + x + 1$

$= 2 + 2x = 0$