Define the product

$$\alpha\beta = \left(\sum_{g\in G} a_g g\right)\left(\sum_{h\in G} b_h h\right) = \sum_{g,h\in G} a_g b_h gh$$

Notes :

(1) We can also write the product $\alpha\beta$ as $\sum_{u\in G} C_u u$, where $C_u = \sum_{gh=u} a_g b_h$

(2) RG is a ring (with addition and multiplication defined as above).

(3) Given $\alpha \in RG$ and $\lambda \in R$, we can define a multiplication

$$\lambda.\alpha = \lambda \sum_{g\in G} a_g g = \sum_{g\in G} (\lambda a_g) g.$$

(4) RG is an example of a hypercomplex number system (if $R = \mathbb{R}$).

Definition 1.12 *Let R be a ring. An abelian group $(M, +)$ is called a* ***(left) R-module*** *if for each $a, b \in R$ and $m \in M$, we have a product $am \in M$ such that*

(i) $(a + b)m = am + bm$

(ii) $a(m_1 + m_2) = am_1 + am_2$

(iii) $a(bm) = (ab)m$

(iv) $1.m = m \;\forall\, a, b \in R$ *and* $\forall\, m, m_1, m_2 \in M$.

Similarly we could define a ***(right) R-module***

(i) $m(a + b) = ma + mb$

(ii) $(m_1 + m_2)a = m_1 a + am_2 a$

(iii) $m(ab) = (ma)b$

(iv) $m.1 = m \;\forall\, a, b \in R$ *and* $\forall\, m, m_1, m_2 \in M$.