Chapter 1

Introduction

1.1 Definitions and examples of Rings and Group Rings

Definition 1.1 *A **ring** is a set R with two binary operations $+$ and $\cdot$ such that*

(i) $a + (b + c) = (a + b) + c$

(ii) $\exists\, 0 \in R$ s.t. $a + 0 = a = 0 + a$

(iii) $\exists - a \in R$ s.t. $a + (-a) = 0 = (-a) + a$

(iv) $a + b = b + a$

(v) $a.(b.c) = (a.b).c$

(vi) $a.(b + c) = a.b + b.c$

(vii) $(a + b).c = a.c + b.c \quad \forall\, a, b, c \in R$

Definition 1.2 *If $a.b = b.a\ \forall\, a, b \in R$, then R is a **commutative ring**.*

Example 1.3 *$(\mathbb{Z}, +, \cdot)$ is a commutative ring.*

Example 1.4 *The set P of polynomials of any degree over $\mathbb{R}$ is a ring (with the obvious multiplication and addition). This is also a commutative ring e.g. $(2x^2 + 1)(3x + 2) = (3x + 2)(2x^2 + 1) \in P$.*