

Writing the elements in lexicographical order :

$$\begin{aligned} &0 + 0.x, 0 + 1.x, 0 + 2.x \\ &1 + 0.x, 1 + 1.x, 1 + 2.x \\ &2 + 0.x, 2 + 1.x, 2 + 2.x \end{aligned}$$

$$\mathbb{F}_3C_2 = \{0, 1, 2, x, 2x, 1 + x, 1 + 2x, 2 + x, 2 + 2x\}.$$

$$\varepsilon : \mathbb{F}_3C_2 \longrightarrow \mathbb{F}_3$$

$\varepsilon(\alpha)$	$\alpha \in \mathbb{F}_3C_2$
0	$\{0, 2 + x, 1 + 2x\}$
1	$\{1, x, 2 + 2x\}$
2	$\{2, 2x, 1 + x\}$

$\mathcal{U}(\mathbb{F}_3C_2) = \{1, x, 2, 2x\}$ , since  $1^2 = 1$ ,  $x^2 = 1$ ,  $2^2 = 1$  and  $(2x)^2 = 1$ . In a group inverses are unique, so we don't need to multiply these anymore.  $\mathcal{U}(\mathbb{F}_3C_2) \cong C_2 \times C_2$  since it has no elements of order 4, so  $\mathcal{U}(\mathbb{F}_3C_2) \not\cong C_4$ .

$(1+x)(1+x) = 1 + x + x + x^2 = 2 + 2x \neq 1$ .  $(1+x)(2+x) = 2 + x + 2x + x^2 = 0 \neq 1$ . Note that these are zero divisors so they are not units. Also  $(1+2x)(1+2x) = 1 + 2x + 2x + 4x^2 = 2 + x$  and  $(1+2x)(2+2x) = 2 + 2x + 4x + 4x^2 = 0$ .

$$\therefore \mathcal{ZD}(\mathbb{F}_3C_2) = \{1 + x, 2 + x, 1 + 2x, 2 + 2x\}$$

$$\text{Note } \mathbb{F}_3C_2 = \mathcal{U}(\mathbb{F}_3C_2) \cup \mathcal{ZD}(\mathbb{F}_3C_2) \cup \{0\}.$$

**Conjecture 2.4** *In general in any group ring  $RG$ , do we have*

$$\mathbb{F}_3C_2 = \mathcal{U}(\mathbb{F}_3C_2) \cup \mathcal{ZD}(\mathbb{F}_3C_2) \cup \{0\}$$

**Lemma 2.5** *Let  $I$  be an ideal of a ring  $R$ , with  $I \neq R$ . Then  $I$  contains no invertible elements.*