Chapter 2

Ideals And Homomorphisms of RG

Let R be a ring (usually commuatative) and G a group. Then RG is a group ring (defined before). Since RG is a ring, we can talk about ideals of G, ring homomorphisms of RG and factor groups of RG.

Definition 2.1 *Consider the function* $\varepsilon : RG \longrightarrow R$ *defined by* $\varepsilon\left(\sum_{g\in G} a_g g\right) = \sum_{g\in G} a_g$. *This function is called the* ***augmentation map***. ε *maps RG onto R.*

Let $r \in R$ then $\varepsilon(r.1) = r$ (onto). Let $rg \in RG$ and $rh \in RG$, the $\varepsilon(rg) = \varepsilon(rh) = r$. However $rg \neq rh$, thus ε is not one-to-one. ε is a ring homomorphism from RG onto R (an epimorphism). Let $\alpha = \sum_{g\in G} a_g g$ and $\beta = \sum_{g\in G} b_g g$ where $\alpha, \beta \in RG$. Then

$$\varepsilon(\alpha + \beta) = \varepsilon\left(\sum_{g\in G}(a_g + b_g)g\right) = \sum_{g\in G}(a_g + b_g) = \sum_{g\in G} a_g + \sum_{g\in G} b_g = \varepsilon(\alpha) + \varepsilon(\beta)$$