

$$\begin{array}{ccc}
 R & \xrightarrow{\omega} & R/I \\
 \downarrow & & \downarrow \\
 J & \xrightarrow{\omega} & J/I \\
 \downarrow & & \downarrow \\
 \omega^{-1}(\mathfrak{I}) = J_1 & \xrightarrow{\omega} & J_1/I = \mathfrak{I} \\
 \downarrow & & \downarrow \\
 \text{Ker}(\omega) = I & \xrightarrow{\omega} & I/I = \{0\} \\
 \downarrow & & \\
 \{0\} & &
 \end{array}$$

Note that a ring homomorphism preserves subsets and ideal.

Theorem 1.51 (2nd Isomorphism Theorem)

$$\begin{array}{c}
 R \\
 \downarrow \\
 I + J \\
 \swarrow \quad \searrow \\
 I \quad \quad J \\
 \swarrow \quad \searrow \\
 I \cap J \\
 \downarrow \\
 \{0\}
 \end{array}
 \quad
 \frac{I+J}{I} \cong \frac{J}{I \cap J} \quad \text{also} \quad \frac{I+J}{J} \cong \frac{I}{I \cap J}$$