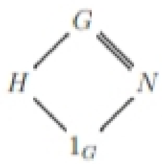
1.3 Isomorphism Theorems

Theorem 1.49 *(1^{st} Isomorphism theorem for groups) Let $f \rightarrowtail S$. Then $G/N \cong S$ where $N = Ker(f)$.*

For rings , the kernal is an ideal. Let G be a group, $H \lhd G$ and $N \unlhd G$. Then



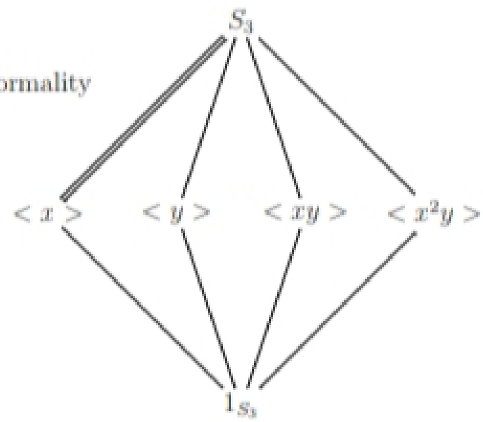
$G/N \cong H/1$

also

$G/H \cong N/1$

Example 1.50 *$S_3 =< x, y \,|\, x^3 = y^2 = 1, yxy = x^2 >$. Let's construct a lattice diagram of subgroups*

Double lines means normality



Now consider $\omega : R \longrightarrow R/I$ where $\omega(r) = r + I$ (the cononical projection). Let $J \supseteq I$, then $\omega(J) = \{j + I : j \in J\} = J/I \subset R/I$. J/I is not only a subset, it is also an ideal of R/I i.e. $J/I \lhd R/I$.