isomorphic as rings.

1.2 Ring Homomorphisms and Ideals

Lemma 1.37 *Let $f : R \longrightarrow S$ be a ring homomorphism, then*

(i) $f(0_r) = 0_s$.

(ii) $f(-a) = -f(a)$.

Proof. (i) Take $a \in R$. $f(a) = f(a + 0_r) = f(a) + f(0_r)$. Thus $f(a) = f(a) + f(0) = f(0) + f(a)\ \forall\, a \in R$. So

$$\begin{aligned} -f(a) + f(a) &= 0_s \\ &= -f(a) + (f(a) + f(0_r)) \\ &= (-f(a) + f(a) + f(0_r) \\ &= 0_s + f(0_r) = f(0_r) \\ &= 0_s \end{aligned}$$

$$\therefore f(0_r) = 0_s$$

(ii) $f(a + (-a)) = f(0_r) = 0_s = f(a) + f(-a)$

$$\therefore f(-a) = -f(a)$$

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Definition 1.38 *Let L be a subset of the ring R. L is called a* ***left ideal*** *of R if*

(i) $x, y \in L \Longrightarrow x - y \in L$.

(ii) $x \in L,\ a \in R \Longrightarrow ax \in L$ *(left multiplication by an element of R).*

$$\therefore R.L = L$$