Clearly $(\mathbb{F}_2C_2, \cdot)$ is not a group (since $0.a = 0 \; \forall \; a \in \mathbb{F}_2C_2$). Also $(\mathbb{F}_2C_2 \setminus \{0\}, \cdot)$ does not form a group (since $(1+x)^2 = 0$ and 0 is not an element of $\mathbb{F}_2C_2 \setminus \{0\}$.

Note : that the unit group of $\mathbb{F}_2C_2$ is $\{1, x\}$.

$\underline{\mathcal{U}(\mathbb{F}_2C_2)}$

$\mathcal{U}(\mathbb{F}_2C_2) = \{1, x\} \cong C_2$

$\cdot$	1	x
1	1	x
x	x	1

Conjecture 1.33 $\mathcal{U}(RG) = G$.

Note that G is isomorphic (as a group) to a subgroup of $\mathcal{U}(RG)$ via the embedding

$$\theta : G \hookrightarrow \mathcal{U}(RG) \quad g \mapsto 1.g$$

We often associate G with $\theta(G) < \mathcal{U}(RG)$ and abusing the notation, we write $G < \mathcal{U}(RG)$.

Recall that in $\mathbb{F}_2C_2$, $(1+x)^2 = 0$. So $1+x$ is the only zero divisor of $\mathbb{F}_2C_2$.

Conjecture 1.34 $RG = \{0\} \cup \mathcal{U}(RG) \cup \mathcal{ZD}(RG)$ *(where* $\mathcal{ZD}(RG)$ *are the zero divisors of* G.

Consider (1) $\mathbb{F}_3C_2$ and (2) $\mathbb{F}_2C_3$.

(1) $\mathbb{F}_3C_2$
$\mathbb{F}_3C_2 = \{a.1 + b.x \,|\, a, b \in \mathbb{F}_3\}$. There are 3 choices for $a \in \{0, 1, 2\}$ and there are 3 choices for $b \in \{0, 1, 2\}$ so there are $3.3 = 9$ elements in $\mathbb{F}_3C_2$.

(2) $\mathbb{F}_2C_3$