

1.3. Tautology / Contradiction / Contingency

Definition 1.3.1. (Tautology)

A tautology (theorem or lemma) is a logical proposition that is always true.

Remark 1.3.2. One informal way to check whether or not a certain logical formula is a theorem is to construct its truth table.

Example 1.3.3. $p \vee \sim p$.

Definition 1.3.4. (Contradiction)

A contradiction is a logical proposition that is always false.

Example 1.3.5. $p \wedge \sim p$.

Definition 1.3.6. (Contingency)

A contingency is a logical proposition that is neither a tautology nor a contradiction.

Example 1.3.7.

(i) The logical proposition $p \vee q \rightarrow \sim r$ is a contingency. See Example 1.2.3(i).

(ii) The logical proposition $p \vee \sim (p \wedge q)$ is a tautology.

p	q	$p \wedge q$	$\sim (p \wedge q)$	$p \vee \sim (p \wedge q)$
T	T	T	F	T
T	F	F	T	T
F	T	F	T	T
F	F	F	T	T

Exercise 1. 1.3.8

(i) Build a truth table to verify that the logical proposition

$$(p \leftrightarrow q) \wedge (\sim p \wedge q)$$

is a contradiction.