Thus $(x/2)^2 = uv$. But u, v are coprime, for if they had a common divisor d, then $d \mid u - v$ and $d \mid u + v$; that is, ie $d \mid y$, $d \mid z$, which is impossible since y, z do not have a common factor. Therefore, u, v are both perfect squares, so let $u = s^2, v = t^2$, giving the required formulae for x, y, z.

The requirement that $\gcd(y, z) = 1$ implies that $\gcd(s, t) = 1$. Also note that if s, t were both odd or both even, then y, z would both be even, which is impossible for a PPT. Thus, s, t have opposite parity. □

Theorem 15.4. *The radius of the inscribed circle of a Pythagorean triangle, that is, one whose sides are a PT, is always an integer.*

Proof Let r be the radius of the circle, inscribed in a triangle whose sides are x, y, z satisfying $x^2 + y^2 = z^2$ for integer x, y, z. Joining each corner to the circumcentre, we have three triangles whose total area is $rx/2 + ry/2 + rz/2$; this is actually the area of the original triangle, $xy/2$, so we get $xy = r(x + y + z)$. Since any integral solutions for x, y, z can be written as $x = 2kst, y = k(s^2 - t^2), z = k(s^2 + t^2)$, we get $2k^2st(s^2 - t^2) = r(2st + 2s^2)$, giving $r = kt(s - t)$, an integer. □