(ii) This is a special case of (i).

(iii) We have that

$$\begin{aligned}
&x \equiv y \,(\mathrm{mod}\; n_i)\; \forall i \in \{1,\, 2,\, \ldots,\, r\} \\
\Leftrightarrow\, & n_i \,|\, x - y \,\forall i \in \{1,\, 2,\, \ldots,\, r\} \\
\Leftrightarrow\, & [n_1,\, n_2,\, \ldots,\, n_r] \,|\, x - y \\
\Leftrightarrow\, & x \equiv y \,(\bmod\; [n_1,\, n_2,\, \ldots,\, n_r])\,.
\end{aligned}$$

□

Corollary 12.14. *Take $n \in \mathbb{N}$. If $a_0,\, a_1,\, \ldots,\, a_{n-1}$ is a CSR modulo n, then so is*

$$\lambda a_0,\, \lambda a_1,\, \ldots,\, \lambda a_{n-1}$$

for each $\lambda \in \mathbb{Z}$ such that $(\lambda,\, n) = 1$.

Proof This follows from Theorem 12.13 (ii). □

Example 12.15. We have that 0, 1, 2, 3, 4, 5 is a CSR modulo 6. Since (5, 6) = 1,

$$0, \quad 5, \quad 10\;(\equiv 4\,(\mathrm{mod}\;6))\,, \quad 15\;(\equiv 3\,(\mathrm{mod}\;6))\,, \quad 20\;(\equiv 2\,(\mathrm{mod}\;6))\,, \quad 25\;(\equiv 1\,(\mathrm{mod}\;6))$$

is also a CSR modulo 6. Since (3, 6) = 3,

$$0, \quad 3, \quad 6\;(\equiv 0\,(\mathrm{mod}\;6))\,, \quad 9\;(\equiv 3\,(\mathrm{mod}\;6))\,, \quad 12\;(\equiv 0\,(\mathrm{mod}\;6))\,, \quad 15\;(\equiv 3\,(\mathrm{mod}\;6))$$

is *not* a CSR modulo 6.