Definition 11.13. A *perfect number* is a natural number which is equal to one half of the sum of its positive divisors (or, equivalently, a natural number which is equal to the sum of its positive divisors which are less than itself).

Example 11.14. 6 is a perfect number:

$$6 = \frac{1}{2}(1 + 2 + 3 + 6) \Leftrightarrow 6 = 1 + 2 + 3.$$

Remark 11.15. Only 47 even perfect numbers are known at present. Examples are 6, 28, 496. No odd perfect number has ever been found, but such an occurrence has not been shown to be impossible. However, an odd perfect number would have to satisfy a number of conditions, not least that it would be greater than 10^{300}.

Perfect numbers are closely related to Mersenne primes, as shown by the following result.

Theorem 11.16. *n is an even perfect number if and only if* $n = 2^{k-1}(2^k - 1)$, *where* $2^k - 1$ *is a Mersenne prime.*

The Sieve of Eratosthenes (276 to 194 BC)

To decide if n is prime, it is only necessary to establish if it has any prime divisors p with $p < \sqrt{n}$. Hence to obtain a list of the primes between 2 and n, one should write down all the numbers between 2 and n. One should then cross out all multiples of 2, then cross out all multiples of 3, then cross out all multiples of 5, up to and including all multiples of the largest prime p such that $p < \sqrt{n}$.

The numbers which remain are the primes between 2 and n.

Example 11.17. Consider $n = 20$.

TO BE FILLED IN DURING LECTURES