Peano Axioms for $\mathbb{N}$

(1) $x \in \mathbb{N} \Rightarrow x^* \in \mathbb{N}$, the *successor* of x;

(2) $\exists 1 \in \mathbb{N}$ such that $\nexists a \in \mathbb{N}$ with $a^* = 1$;

(3) for $x, y \in \mathbb{N}$, $x^* = y^* \Rightarrow x = y$;

(4) if S is a subset of $\mathbb{N}$ such that

$$\text{(a) } 1 \in S, \quad \text{(b) } x \in S \Rightarrow x^* \in S,$$

then $S = \mathbb{N}$.