Remark 7.24. The Cayley embedding depends on the ordering of the elements of G.

Indeed, consider again the group $C_3 = \{1, \omega, \omega^2\}$ of cube roots of unity, and take $g_1 = 1$, $g_2 = \omega^2$, $g_3 = \omega$. Then the mapping $\varphi : C_3 \mapsto S_3$ satisfies

$$C_3 \ni 1 \mapsto \begin{pmatrix} 1 & \omega^2 & \omega \\ 1 & \omega^2 & \omega \end{pmatrix}, \quad \text{i.e.} \begin{pmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \end{pmatrix};$$
$$C_3 \ni \omega^2 \mapsto \begin{pmatrix} 1 & \omega^2 & \omega \\ \omega^2 & \omega & 1 \end{pmatrix}, \text{i.e.} \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix};$$
$$C_3 \ni \omega \mapsto \begin{pmatrix} 1 & \omega^2 & \omega \\ \omega & 1 & \omega^2 \end{pmatrix}, \text{i.e.} \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}.$$