

Let

α = rotation anti-clockwise through $\frac{2\pi}{3} = 120^\circ$,

β_1 = reflection about the $Re(z)$ axis,

β_2 = reflection about the line joining 0 to $e^{\frac{2\pi i}{3}}$,

β_3 = reflection about the line joining 0 to $e^{\frac{4\pi i}{3}}$.

Then $D_3 = \{1, \alpha, \alpha^2, \beta_1, \beta_2, \beta_3\}$ and

$$\alpha = (ABC), \quad \alpha^2 = (ACB), \quad \beta_1 = (BC), \quad \beta_2 = (AC), \quad \beta_3 = (AB).$$

The multiplication table of D_3 is

	1	α	α^2	β_1	β_2	β_3
1	1	α	α^2	β_1	β_2	β_3
α	α	α^2	1	β_2	β_3	β_1
α^2	α^2	1	α	β_3	β_1	β_2
β_1	β_1	β_3	β_2	1	α^2	α
β_2	β_2	β_1	β_3	α	1	α^2
β_3	β_3	β_2	β_1	α^2	α	1

For example, under $\alpha\beta_1$ we have that $A \mapsto B \mapsto C$, $B \mapsto C \mapsto B$, $C \mapsto A \mapsto A$.

2. Let P_n be a regular n -gon and D_n be its group of symmetries. Then D_n is called the *dihedral group of degree n* and has order $2n$:

$$D_n = \underbrace{\{1, \alpha, \alpha^2, \dots, \alpha^{n-1}\}}_{\text{rotations}}, \underbrace{\{\beta_1, \beta_2, \dots, \beta_n\}}_{\text{reflections}};$$

where α is a rotation through $\frac{2\pi}{n}$ about the centre of P_n and $\beta_1, \beta_2, \dots, \beta_n$ are reflections about the axes of symmetry of P_n .

If n is even, then an axis of symmetry either joins two opposite vertices or the mid-points of two opposite sides.

If n is odd, then an axis of symmetry joins a vertex to the mid-point of the opposite side.