We have that

**Proposition 6.3.** *Let $G$ be a group of invertible mappings from a set $X$ to itself, and take $Y \subset X$. The set of mappings in $G$ under which $Y$ is invariant is indeed a subgroup of $G$ under the operation of composition of mappings.*

**Proof** This is an exercise. □

**Example 6.4.** Take $X = \{1, 2, 3, 4\}$. Then $S(X) = S_4$. Consider the subgroup

$$G = \{i, (1\,2)(3\,4), (1\,3)(2\,4), (1\,4)(2\,3)\}$$

of $S(X)$. Then

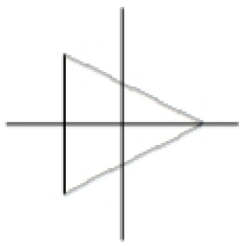
- the subgroup of $G$ under which the subset $Y = \{1, 2\}$ of $X$ is invariant is $\{i, (1\,2)(3\,4)\}$;
- the subgroup of $G$ under which the subset $Y = \{1, 3\}$ of $X$ is invariant is $\{i, (1\,3)(2\,4)\}$;
- the subgroup of $G$ under which the subset $Y = \{1, 2, 3\}$ of $X$ is invariant is $\{i\}$;
- the subgroup of $G$ under which the subset $Y = \{1, 2, 3, 4\}$ of $X$ is invariant is $G$ itself.

**Example 6.5.** Consider the set $\mathbb{R}^2$ and take $G$ to be the subgroup of $S(\mathbb{R}^2)$ consisting of all the *isometries* of $\mathbb{R}^2$, i.e. consisting of all the invertible mappings

$$\psi : \mathbb{R}^2 \mapsto \mathbb{R}^2$$

which leave distance unchanged.

Now consider the subset $Y$ of $\mathbb{R}^2$ given by the equilateral triangle in the complex plane centered at the origin with vertices in the positions corresponding to the cube roots of unity:



The group of isometries under which this triangle is invariant is denoted by $D_3$.

**Examples 6.6.** 1. Consider again the equilateral triangle in the complex plane centered at the origin with vertices in the positions corresponding to the cube roots of unity.