(ii) This can be proved by multiplying on the right by a^{-1} and arguing similarly to as in (i).

□

Remark 1.10. A consequence of the Cancellation Law 1.9 is that every element of a finite group appears exactly once in each row and exactly once in each column of the product table. So each row and each column of the product table consists of some ordering of the elements of the group.

Hence any product table which does not satisfy this condition is *not* the product table of a finite group.

However, this condition is *not* sufficient to ensure that the product table is one of a group. Indeed, it is still necessary to check that the associativity, inverse and identity axioms are satisfied.

Example 1.11. Consider the product on the set

$$G = \{a,\ b,\ c,\ d,\ e,\ f\}$$

defined by the product table

	a	b	c	d	e	f
a	a	b	c	d	e	f
b	b	a	e	f	c	d
c	c	f	a	e	d	b
d	d	e	f	a	b	c
e	e	d	b	c	f	a
f	f	c	d	b	a	e

(1.1)

(i.e. $bc = e$, $cb = f$, etc.). Note that it satisfies the criterion of Remark 1.10 that every element of G appears exactly once in each row and exactly once in each column.

To verify that the product is associate, we have to check the required equality holds for each of the $6 \times 6 \times 6$ possible triples of elements of G in turn. Two of the required equalities are

$$a\,(bc) = ae = e = bc = (ab)\,c \quad a\,(bd) = af = f = bd = (ab)\,d.$$

Furthermore, we see that $a \in G$ behaves like an identity since

$$aa = a = aa, \quad ba = b = ab, \quad ca = c = ac, \quad da = d = ad, \quad ea = e = ae, \quad fa = f = af.$$

Finally, we check that an operation of inverse must be found. Indeed, since

$$dd = cc = bb = aa = a, \quad ef = a = fe,$$

we have that a, b, c, d, f, e behave like the inverses of a, b, c, d, e, f respectively.