Hence

$$b = a^{-1}(ab) = \underbrace{a^{-1}1}_{\text{by identity axiom}} = a^{-1}.$$

(ii) Suppose that $e \in G$ satisfies

$$ae = a = ea \quad \forall\, a \in G.$$

We need to show that $e = 1$. Indeed, considering $a = 1$ gives that $1e = 1 = e1$. By the identity axiom, $1e = e$. Hence $e = 1$.

□

For a finite group $G$, its product table can be used to determine the identity element and the inverse of each element.

**Proposition 1.8.** *For any group $G$, one has that*

(i) $(ab)^{-1} = b^{-1}a^{-1} \quad \forall\, a, b \in G,$

(ii) $(a^{-1})^{-1} = a \quad \forall\, a \in G.$

**Proof** This is left as an exercise.
*Hint: in each case, check that the element on the right behaves like the stated inverse and use the uniqueness of inverses.* □

**Proposition 1.9** (the Cancellation Law)**.** *For any group $G$ and elements $a$, $b$, $x \in G$, one has that*

(i) $ax = ay \Rightarrow x = y,$

(ii) $xa = ya \Rightarrow x = y.$

**Proof** (i) Suppose that $a$, $b$, $x \in G$ satisfy $ax = ay$.

Multiplying on the left by $a^{-1}$ gives that $a^{-1}(ax) = a^{-1}(ay)$. Furthermore,

$$a^{-1}(ax) \underset{\substack{\uparrow\\ \text{by associativity}\\ \text{axiom}}}{=} (a^{-1}a)x \underset{\substack{\uparrow\\ \text{by inverse}\\ \text{axiom}}}{=} 1x \underset{\substack{\uparrow\\ \text{by identity}\\ \text{axiom}}}{=} x,$$

and similarly, $a^{-1}(ay) = y$. Hence $x = y$.