Examples 1.4. (1) Consider the set $\mathbb{R}^* = \mathbb{R} \setminus \{0\}$ of non-zero real numbers, together with operations of product, inverse and identity given by multiplication, reciprocal and the number one respectively. The axioms (i), (ii), (iii) and (iv) can be verified by using the properties of the real numbers. Hence it follows that $\mathbb{R}^*$ is a group under the given operations.

The set $\mathbb{Q}^* = \mathbb{Q} \setminus \{0\}$ of non-zero rational numbers is a group under the same operations.

The set $\mathbb{C}^* = \mathbb{C} \setminus \{0\}$ of non-zero complex numbers together with operations of product, inverse and identity given by complex multiplication, complex inversion and the number $1 \in \mathbb{C}^*$ respectively is a group.

(2) The set $\mathrm{GL}(2, \mathbb{R})$ of invertible 2×2 matrices over the real numbers together with operations of product, inverse and identity given by matrix multiplication, matrix inversion and the 2×2 identity matrix $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ respectively is a group.

For $n \geq 2$, the sets $\mathrm{GL}(n, \mathbb{R})$ and $\mathrm{GL}(n, \mathbb{Q})$ of invertible $n \times n$ matrices over the real and rational numbers respectively together with operations of product, inverse and identity given by matrix multiplication, matrix inversion and the $n \times n$ identity matrix

$$\underbrace{\begin{pmatrix} 1 & 0 & 0 & \cdots & 0 & 0 & 0 \\ 0 & 1 & 0 & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 0 & 1 & 0 \\ 0 & 0 & 0 & \cdots & 0 & 0 & 1 \end{pmatrix}}_{n} \Bigg\} n$$

respectively are groups.

For $n \geq 2$, the set $\mathrm{GL}(n, \mathbb{C})$ of invertible $n \times n$ matrices over the complex numbers together with operations of product, inverse and identity given by complex matrix multiplication, complex matrix inversion and the $n \times n$ identity matrix respectively is a group.

(3) Let n be a positive integer and take

$$\omega = \cos\left(\frac{2\pi}{n}\right) + \mathrm{i}\sin\left(\frac{2\pi}{n}\right) \in \mathbb{C}.$$

The set

$$C_n = \left\{1, \omega, \omega^2, \ldots, \omega^{n-1}\right\}$$

together with the operations of product, inverse and identity given by complex multiplication, complex inversion and the number $1 \in C_n$ respectively is a group. In fact, C_n is the group of the n^{th} *roots of unity*.