It is worth noting that the previous proof also allows us to locate the inverses of a regular element.

Lemma 8.11. *If $a \in S$ is regular, and $x \in V(a)$, then there exist idempotents $e = ax$ and $f = xa$ such that*

$$a \,\mathcal{R}\, e \,\mathcal{L}\, x, \;\; a \,\mathcal{L}\, f \,\mathcal{R}\, x.$$

Conversely, if $a \in S$ and e, f are idempotents such that

$$a \,\mathcal{R}\, e, \;\; a \,\mathcal{L}\, f,$$

then there exists $x \in V(a)$ such that $ax = e$ and $xa = f$ (and then

$$e \,\mathcal{L}\, x, \;\; f \,\mathcal{R}\, x.)$$

a		$e = ax$
$f = xa$		x

Proof. For the first part, one just has to define $e = ax$ and $f = xa$. As we have seen, e and f are idempotents satisfying the required properties.
The converse follows directly from the proof of Corollary 8.10 (Corollary to Green's Lemmas). □

Example 8.12.

(1) For $\mathcal{M}^0 = \mathcal{M}^0(G; I; \Lambda; P)$ we know that $\mathcal{M}^0 \setminus \{0\}$ is a $\mathcal{D}$-class. We have $H_{i\lambda} = \{(i, g, \lambda) \mid g \in G\}$. If $p_{\lambda i} \neq 0$, $H_{i\lambda}$ is a group $\mathcal{H}$-class. If $p_{\lambda i}, p_{\mu j} \neq 0$ then $H_{i\lambda} \cong H_{j\mu}$ (already seen directly).
(2) The Bicyclic Monoid B is bisimple with $E(B) = \{(a, a) \mid a \in \mathbb{N}^0\}$ and $H_{(a,a)} = \{(a, a)\}$. Clearly $H_{(a,a)} \cong H_{(b,b)}$.
(3) In $\mathcal{T}_n$, then $\alpha \,\mathcal{D}\, \beta \Leftrightarrow \rho(\alpha) = \rho(\beta)$ where $\rho(\alpha) = |\operatorname{Im}(\alpha)|$. By Corollary 8.10 , if $\varepsilon, \mu \in E(\mathcal{T}_n)$ and $\rho(\varepsilon) = \rho(\mu) = m$ say, then $H_\varepsilon \cong H_\mu$. In fact $H_\varepsilon \cong H_\mu \cong \mathcal{S}_m$.