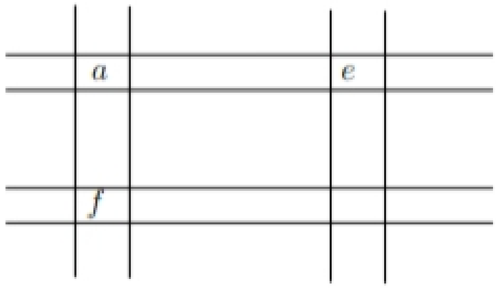


$$ax = afse = ase = e^2 = e$$

and so $a = ea = axa$. Since $a \mathcal{L} f$ there exists $t \in S^1$ with $ta = f$. Then

$$xa = fsea = fsa = tasa = ta = f.$$

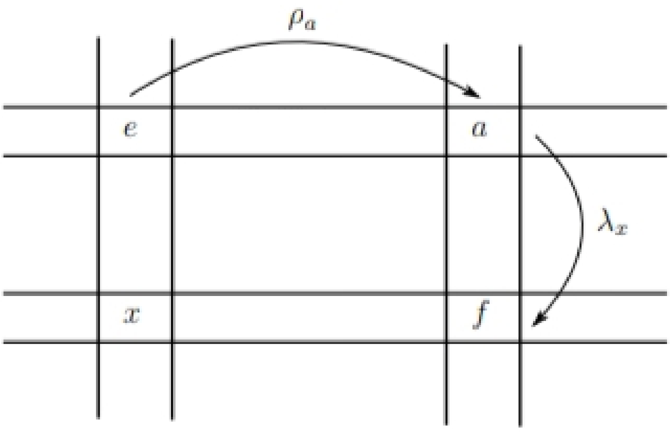
Also

$$xax = fx = ffse = fse = x.$$

So we have

$$e = ax \qquad a = axa \qquad x = xax \qquad f = xa.$$

We have $e \mathcal{R} a$ and $ea = a$ therefore $\rho_a : H_e \to H_a$ is a bijection. From $a \mathcal{L} f$ and $xa = f$ we have $\lambda_x : H_a \to H_f$ is a bijection. Hence $\rho_a \lambda_x : H_e \to H_f$ is a bijection.
So we have the diagram



Let $h, k \in H_e$. Then

$$h(\rho_a\lambda_x)k(\rho_a\lambda_x) = (xha)(xka) = xh(ax)ka =$$
$$xheka = xhka = hk(\rho_a\lambda_x).$$

So, $\rho_a\lambda_x$ is an isomorphism and $H_e \cong H_f$. □