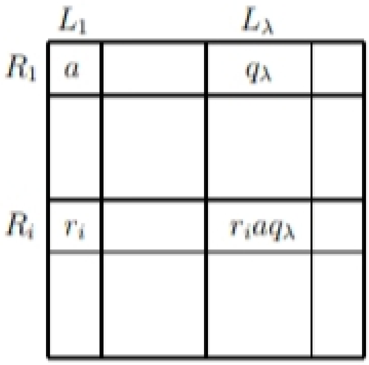

So, $P = (p_{\lambda i}) = (q_\lambda r_i)$ is a $\Lambda \times I$ matrix over $G \cup \{0\}$. For any $i \in I$, by the 0-simple Lemma (Lemma 7.15) we have $Sr_iS = S$. So, $ur_iv \neq 0$ for some $u, v \in S$. Say, $u = r_k b q_\lambda$ for some k, λ and b. Then

$$p_{\lambda i} = q_\lambda r_i \neq 0$$

as $r_k b q_\lambda r_i v \neq 0$. Therefore every column of P has a non-zero entry. Dually for rows. Therefore

$$\mathcal{M}^0 = \mathcal{M}^0(G; I; \Lambda; P)$$

is a Rees Matrix Semigroup over a group G. For any $x \in \mathcal{M}^0$ ($x = 0$ or x is a triple) then

$$(0x)\theta = 0\theta = 0 = 0(x\theta) = 0\theta x\theta.$$

Also, $(x0)\theta = x\theta 0\theta$. For $(i, a, \lambda), (k, b, \mu) \in \mathcal{M}^0$ we have

$$\begin{aligned} ((i,a,\lambda)(k,b,\mu))\theta &= \begin{cases} 0\theta & \text{if } p_{\lambda k} = 0, \\ (i, a p_{\lambda k} b, \mu)\theta & \text{if } p_{\lambda k} \neq 0, \end{cases} \\ &= \begin{cases} 0 & \text{if } p_{\lambda k} = 0, \\ r_i a p_{\lambda k} b q_\mu & \text{if } p_{\lambda k} \neq 0, \end{cases} \\ &= r_i a p_{\lambda k} b q_\mu, \\ &= r_i a q_\lambda r_k b q_\mu, \\ &= (i, a, \lambda)\theta (k, b, \mu)\theta. \end{aligned}$$

Therefore θ is a morphism, and since it is bijective, it is an isomorphism. □

8. Regular Semigroups

Definition 8.1. We say that $a \in S$ is *regular* if $a = axa$ for some $x \in S$. The semigroup S is *regular* if every $a \in S$ is regular.