Proof. If $S \cong \mathcal{M}^0(G; I; \Lambda; P)$ where G is a group, we know $\mathcal{M}^0$ is completely 0-simple (by Proposition 7.3, Rees Matrix facts and Example 7.9), hence S is completely 0-simple.

Conversely, suppose that S is completely 0-simple. By the $\mathcal{D} = \mathcal{J}$ Theorem, $\mathcal{D} = \mathcal{J}$ (as S has M_R and M_L, it must have $(\star)$). As S is 0-simple, the $\mathcal{D} = \mathcal{J}$-classes are $\{0\}$ and $S \setminus \{0\}$. Let $D = S \setminus \{0\}$. By Corollary 7.16, D contains an idempotent $e = e^2$.

Let $\{R_i \mid i \in I\}$ be the set of $\mathcal{R}$-classes in D (so I indexes the non-zero $\mathcal{R}$-classes). Let $\{L_\lambda \mid \lambda \in \Lambda\}$ be the set of $\mathcal{L}$-classes in D (so Λ indexes the non-zero $\mathcal{L}$-classes).

Denote the $\mathcal{H}$-class $R_i \cap L_\lambda$ by $H_{i\lambda}$. Since D contains an idempotent e, D contains the *subgroup* H_e (Maximum Subgroup Theorem or Green's Theorem). Without loss of generality we can assume that both I and Λ contain a special symbol 1, and we can also assume that $e \in H_{11}$. Put $G = H_{11}$, which is a group.

For each $\lambda \in \Lambda$ let us choose and fix an arbitrary $q_\lambda \in H_{1\lambda}$ (take $q_1 = e$).
Similarly, for each $i \in I$ let $r_i \in H_{i1}$ (take $r_1 = e$).
Notice that

$$e = e^2, e \,\mathcal{R}\, q_\lambda \Rightarrow eq_\lambda = q_\lambda$$

Thus, by Green's Lemma,

$$\rho_{q_\lambda} : H_e = G \to H_{1\lambda}$$

is a bijection. Now,

$$e = e^2, e \,\mathcal{L}\, r_i \Rightarrow r_i e = r_i.$$

By the dual of Green's Lemma

$$\lambda_{r_i} : H_{1\lambda} \to H_{i\lambda}$$

is a bijection. Therefore for any $i \in I$, $\lambda \in \Lambda$ we have

$$\rho_{q_\lambda}\lambda_{r_i} : G \to H_{i\lambda}$$

is a bijection.

NOTE. By the definition of ρ_{q_λ} and λ_{r_i}, we have that

$$a\rho_{q_\lambda}\lambda_{r_i} = r_i a q_\lambda$$

for every $a \in G, i \in I$ and $\lambda \in \Lambda$.

So, each element of $H_{i\lambda}$ has a unique expression as $r_i a q_\lambda$ where $a \in G$. Hence the mapping

$$\theta : (I \times G \times \Lambda) \cup \{0\} \to S$$

given by $0\theta = 0$, $(i, a, \lambda)\theta = r_i a q_\lambda$ is a bijection.
Put $p_{\lambda i} = q_\lambda r_i$. If $p_{\lambda i} \neq 0$ then $q_\lambda r_i \,\mathcal{D}\, q_\lambda \,\mathcal{D}\, r_i$. By the rectangular property

$$e \,\mathcal{R}\, q_\lambda \,\mathcal{R}\, q_\lambda r_i \,\mathcal{L}\, r_i \,\mathcal{L}\, e$$

so that $q_\lambda r_i \in G$.