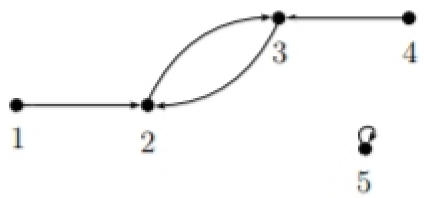


No tails of length $\geqslant 2$. Therefore $\alpha$ lies in a subgroup. Hence $\alpha$ lies in a maximal subgroup. Hence the maximal subgroup containing $\alpha$ is $\mathcal{H}_\alpha$. For any $\beta$

$$\begin{aligned}\beta \in H_\alpha &\Leftrightarrow \beta\, \mathcal{H}\, \alpha, \\ &\Leftrightarrow \beta\, \mathcal{R}\, \alpha \text{ and } \beta\, \mathcal{L}\, \alpha, \\ &\Leftrightarrow \ker \beta = \ker \alpha \text{ and } \operatorname{Im} \beta = \operatorname{Im} \alpha, \\ &\Leftrightarrow \operatorname{Im} \beta = \{2,3,5\} \text{ and } \ker \beta \text{ has classes } \{1,3\}, \{2,4\}, \{5\}.\end{aligned}$$

We now figure out what the elements of $\mathcal{H}_\alpha$ are. We start with the idempotent. We know that the image of the idempotent is $\{2,3,5\}$ and that idempotents are identities on their images. Thus we must have

$$\varepsilon = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ & 2 & 3 & & 5 \end{pmatrix}.$$

We also know that 1 and 3 go to the same place and 2 and 4 go to the same place. Thus we must have

$$\varepsilon = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 2 & 3 & 2 & 5 \end{pmatrix}.$$

We now have what the idempotent is and then the other elements of $\mathcal{H}_\alpha$ are (note that 1 and 3 must have the same images, just as 2 and 4):

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 2 & 3 & 5 \end{pmatrix}$$
$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 5 & 2 & 5 & 2 & 3 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 5 & 3 & 5 & 2 \end{pmatrix}$$
$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 5 & 3 & 5 & 3 & 2 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 5 & 2 & 5 & 3 \end{pmatrix}.$$

These are all 6 elements.

Check $\mathcal{H}_\alpha \simeq S_3$.