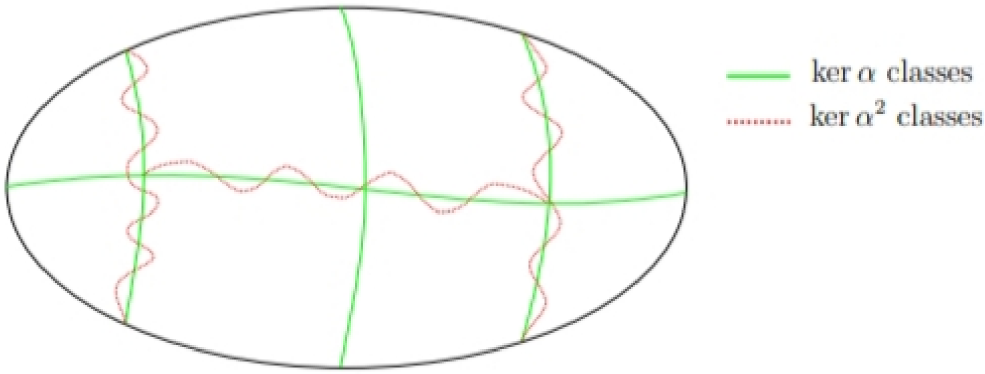


FIGURE 4. The classes of $\ker\alpha$ and $\ker\alpha^2$.

Proof.

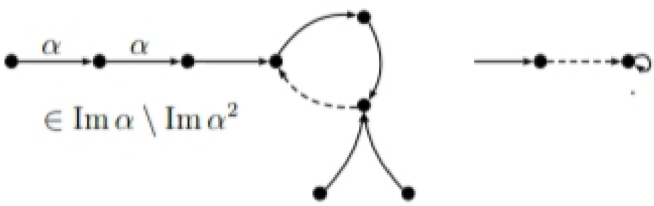
$$\left|\underline{n}/\ker\alpha^2\right| = |\operatorname{Im}\alpha^2| \leq |\operatorname{Im}\alpha| = |\underline{n}/\ker\alpha|\,.$$

Thus $\ker\alpha$ and $\ker\alpha^2$ have the same number of classes if and only if $|\operatorname{Im}\alpha| = |\operatorname{Im}\alpha^2|$. It follows that $\ker\alpha = \ker\alpha^2$ if and only if $\operatorname{Im}\alpha = \operatorname{Im}\alpha^2$. □

We now continue with the proof of Lemma 5.8:

We have that α lies in a subgroup $\Leftrightarrow \operatorname{Im}\alpha = \operatorname{Im}\alpha^2$. Note that elements of $\operatorname{Im}\alpha \setminus \operatorname{Im}\alpha^2$ are exactly those second vertices of tails in the map diagram of α which are not members of a cycle. Thus, $\operatorname{Im}\alpha^2 = \operatorname{Im}\alpha$ if and only if no such vertices exist, thus if and only if all tails have length smaller than or equal to 1. □

An arbitrary element of $\mathcal{T}_n$ looks like:



EXAMPLE 5.9.

(1) We take an element of $\mathcal{T}_5$ to be

$$\alpha = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 1 & 4 & 3 & 1 \end{pmatrix} \in \mathcal{T}_5.$$

This has map diagram