B is not cancellative as e.g.

$$(1,1)(2,2) = (2,2)(2,2).$$

Groups are cancellative (indeed, any subsemigroup of a group is cancellative). $\mathbb{N}^0$ is a cancellative monoid, which is not a group.

DEFINITION 1.15. A zero "0" of a semigroup S is an element such that, for all $a \in S$,

$$0a = a = a0.$$

Adjoining a Zero Let S be a semigroup, then pick a new symbol "0". Let $S^0 = S \cup \{0\}$; define a binary operation $\cdot$ on S^0 by

$$\begin{aligned} a \cdot b &= ab && \text{for all } a \in S,\\ 0 \cdot a = 0 &= a \cdot 0 && \text{for all } a \in S,\\ 0 \cdot 0 &= 0. \end{aligned}$$

Then $\cdot$ is associative, so S^0 is a semigroup with zero 0.

DEFINITION 1.16. S^0 is S *with a zero adjoined.*

2. Standard algebraic tools

DEFINITION 2.1. Let S be a semigroup and $\emptyset \neq T \subseteq S$. Then T is a *subsemigroup* of S if $a, b \in T \Rightarrow ab \in T$. If S is a monoid then T is a *submonoid* of S if T is a subsemigroup and $1 \in T$.

Note T is then itself a semigroup/monoid.

EXAMPLE 2.2. (1) $(\mathbb{N}, +)$ is a subsemigroup of $(\mathbb{Z}, +)$.
(2) $R = \{c_x \mid x \in X\}$ is a subsemigroup of $\mathcal{T}_X$, since

$$c_x c_y = c_y$$

for all $x, y \in X$.
R is a *right zero semigroup* (See Ex.1).
(3) Put $E(B) = \{(a,a) \mid a \in \mathbb{N}^0\}$.
From Ex. 1, $E(S) = \{\alpha \in B : \alpha^2 = \alpha\}$
Claim $E(B)$ is a commutative submonoid of B.
Clearly we have $(0,0) \in E(B)$ and for $(a,a), (b,b) \in E(B)$ we have

$$\begin{aligned} (a,a)(b,b) &= (a - a + t, b - b + t) && \text{where } t = \max\{a, b\},\\ &= (t,t),\\ &= (b,b)(a,a). \end{aligned}$$