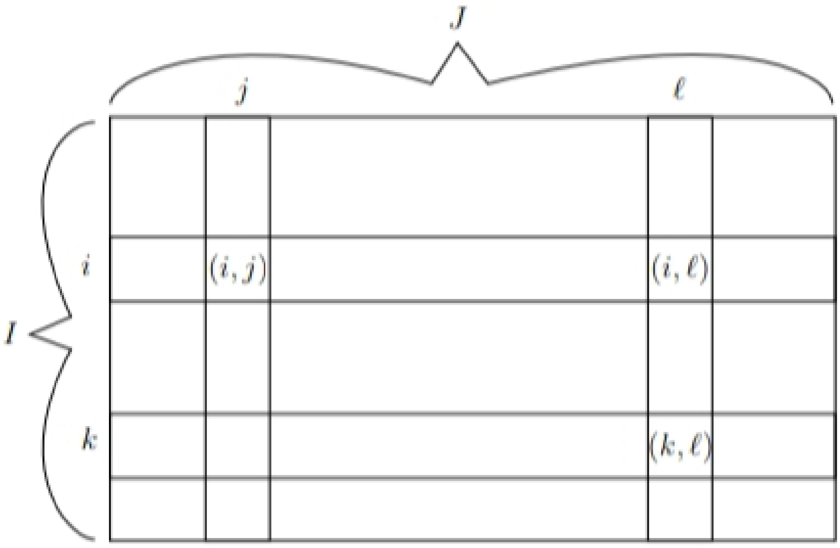

FIGURE 1. The rectangular band.

Then $*$ is associative (check this) so $S \cup \{1\}$ is a monoid with identity 1. Multiplication in $S \cup \{1\}$ extends that in S.
The monoid S^1 is defined by

$$S^1 = \begin{cases} S & \text{if } S \text{ is a monoid,} \\ S \cup \{1\} & \text{if } S \text{ is not a monoid.} \end{cases}$$

DEFINITION 1.8. S^1 is "*S with a 1 adjoined if necessary*".

EXAMPLE 1.9. Let T be the rectangular band on $\{a\} \times \{b, c\}$. Then $T^1 = \{1, (a, b), (a, c)\}$, which has multiplication table

	1	(a,b)	(a,c)
1	1	(a,b)	(a,c)
(a,b)	(a,b)	(a,b)	(a,c)
(a,c)	(a,c)	(a,b)	(a,c)

The Bicyclic Semigroup/Monoid B

If $A \subseteq \mathbb{Z}$, such that $|A| < \infty$, then $\max A$ is the greatest element in A. i.e.

$$\max\{a, b\} = \begin{cases} a & \text{if } a \geqslant b, \\ b & \text{if } b \geqslant a. \end{cases}$$

We note some further things about max: