CHAPTER THREE

AIRCRAFT LANDING PROBLEMS (ALP)

3.4 Techniques to Improve the Solution and Reduce the Computations

In this section we demonstrate two types of methods which are contribute in improving the solution and speed the approach to the good solution. In addition, we will discuss some special cases of ALP.

3.4.1 Time Window Tightening (TWT)

Let Z_{UB} be any upper bound to the problem. Then, it is possible to limit the deviation from target for each plane. Specifically, for plane i, we can update E_i using:

$$E_i = \max\{E_i, T_i - Z_{UB}/g_i\}, \quad i \in P, \qquad \ldots(3.10)$$

Similarly we have

$$L_i = \min\{L_i, T_i + Z_{UB}/h_i\}, \quad i \in P, \qquad \ldots(3.11)$$

The benefit of tightening (closing) the time windows is that (potentially) the sets U and V can be reduced in size, thereby giving a smaller problem to solve.

Example (3.2): The time window tightening of example (3.1) using Eq. (3.10) and (3.11). for instance, Z_{UB}=1060 we have:

$E_i=\max\{E_i,T_i-106\}$ where: $E_1=\max\{129,155-106\}=129$,

$E_2=\max\{195,258-106\}=195$, $E_3=\max\{89,98-35\}=98$.

$L_i=\min\{L_i,T_i+106\}$ where: $L_1=\min\{559,155+106\}=261$,

$L_2=\min\{744,258+106\}=364$, $L_3=\min\{89,98+35\}=133$.

These results are shown in table (3.1).

Table (3.1): time window tightening of example (3.2) for Z_{UB}=1060.

	P_1	P_2	P_3
E_i	129	195	89
T_i	155	258	98
L_i	261	364	133
g_i	10	10	30
h_i	10	10	30

Exercise (3.1): calculate the TWT for:

1. from example (3.1), Z_{UB}=900.
2. from Appendix, for N=10, for 1^{st} 5 aircraft, Z_{UB}=90.
3. from Appendix, for N=15, for 1^{st} 5 aircraft, Z_{UB}=90.

3.4.2 Successive Rules (SR)

Reducing the current sequence is done by using several SR's. When, for each i ($i \in P$), and with its cost given in the objective function (3.9), we can derive SR that restrict the search for an optimal solution. Such rules can be used in some optimization algorithms. These improvements lead to very large decrease in the number of solutions to obtain the optimal solution.

Definition (3.1): Let $W_i=[E_i,L_i]$ be the time window interval of plane $i \in P$, if $W_i \cap W_j=\phi$ (time windows are disjoint) and $L_i<E_j$ we denote for the interval W_i precedes the interval W_j in line number by $W_i \ni W_j$.

Definition (3.2): We say that plane i precedes the plane j (we write $i \to j$ or $(i,j) \in W$) or j precedes the plane i if $W_i \cap W_j=\phi$, for $i \neq j$.

Remark (3.1):

1. $t_i < t_j$ and $t_j \geq t_i+S_{ij}$ if and only if $i \to j$, $\forall i,j \in P$, $i \neq j$.
2. if $E_i \leq E_j \leq L_i$ or $E_i \leq L_j \leq L_i$, then $W_i \cap W_j \neq \phi$ for $i \neq j$, we say that W_i and W_j are overlapped.

Proposition (3.1): if W_i W_j, then $t_i \in W_i < t_j \in W_j$, $\forall i,j \in P$, $i \neq j$.

Proof: since $W_i 3 W_j$, then $t_i \notin W_j$ and $t_j \notin W_i$. Suppose $t_i \geq t_j$, for $t_i = t_j$, $t_j = t_i \in W_i$, C!. For $t_i > t_j$, if $t_j \in W_i$ C!. Take $t_j \notin W_i$. Then $t_j \in$ another interval say W_k, s.t. $W_k 3 W_j$, but $t_j \in W_j$ and that is a contradiction since there is no integer belong to two disjoint intervals in the same time. Then $t_i < t_j$ □

Remark (3.2): if $W_i \cap W_j = \phi$, then $L_i < E_j$ or $L_j < E_i$, $\forall i,j \in P$, $i \neq j$.

Definition (3.3): the $i \rightarrow j$ if one of the following conditions is satisfied:

1. $L_i < E_j$ for $i \neq j$.
2. For $L_i \geq E_j$, if $L_i < E_j + S_{ij}$ for $i \neq j$.

Conditions of SR are shown in figure (5.2).

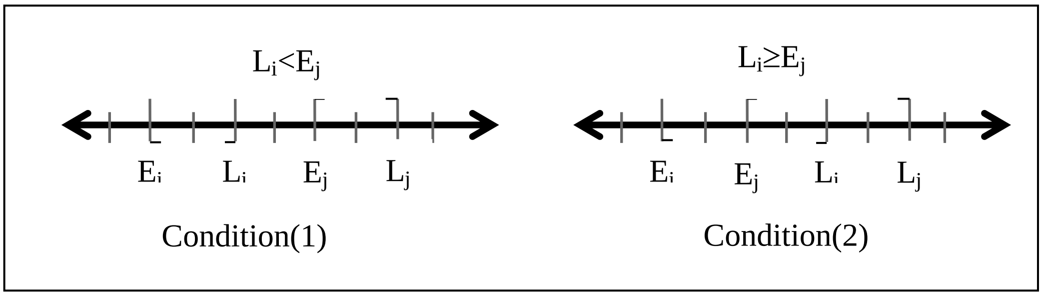

Figure (3.2): Conditions of dominance rules.

Example (3.3): For N=5 let's have the following ALP information:

	P_1	P_2	P_3	P_4	P_5
E_i	129	89	96	111	123
T_i	155	98	106	123	135
L_i	191	110	118	135	147
g_i	10	30	30	30	30
h_i	10	30	30	30	30

S_{ij}	1	2	3	4	5
1	0	15	15	15	15
2	15	0	8	8	8
3	15	8	0	8	8
4	15	8	8	0	8
5	15	8	8	8	0

From definition (3.3), condition (1) we obtain the following SR's:

$2 \rightarrow 1$, $2 \rightarrow 4$, $2 \rightarrow 5$, $3 \rightarrow 1$, $3 \rightarrow 5$.

From condition (2), we have $3 \rightarrow 4$ because of $E_4 + S_{34} = 111 + 8 = 119 > L_3 = 118$, and $4 \rightarrow 1$ because of $E_1 + S_{41} = 129 + 15 = 144 > L_4 = 135$. Figure (3.3) shows the SR's of example (3.3).

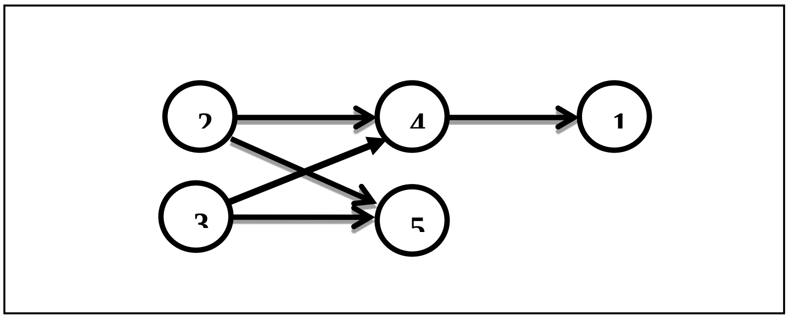

Figure (3.3): Graph of SR of example (3.3).

The adjacency matrix A of the graph shown above is:

$$A = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 & \delta_{15} \\ 1 & 0 & \delta_{23} & 1 & 1 \\ 1 & \delta_{32} & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & \delta_{45} \\ \delta_{51} & 0 & 0 & \delta_{54} & 0 \end{bmatrix} \end{array}$$

Note:

- $\delta_{15}+\delta_{51}=1$, $\delta_{23}+\delta_{32}=1$, $\delta_{45}+\delta_{54}=1$
- the sequencing problem of this ALP can solved by $2^3=8$ possible and no need to try $5!=120$ possible.

Example (3.4): Find the possible sequences for example (3.3):

From adjacency matrix A, we have $(\delta_{15},\delta_{23},\delta_{45})$, 1↔5,2↔3 and 4↔5.

So we have:

i	$(\delta_{15},\delta_{23},\delta_{45})$	Subsequence	sequence
1.	(0,0,0)	5→1,3→2,5→4	3→2→5→4→1
2.	(0,0,1)	5→1,3→2,4→5	3→2→4→5→1
3.	(0,1,0)	5→1,2→3,5→4	2→3→5→4→1
4.	(0,1,1)	5→1,2→3,4→5	2→3→4→5→1
5.	(1,0,0)	1→5,3→2,5→4	3→2→1→5→4
6.	(1,0,1)	1→5,3→2,4→5	3→2→1→4→5
7.	(1,1,0)	1→5,2→3,5→4	2→3→1→5→4

8. (1,1,1) $1\rightarrow5, 2\rightarrow3, 4\rightarrow5$ $2\rightarrow3\rightarrow1\rightarrow4\rightarrow5$

3.4.3 Special Cases

Definition (3.4): Let $S=\max\{S_{ij}\}$, $\forall i,j\in P, i\neq j$, then W_i is called **logical time window** if the length ℓ_i of W_i, for $i\in P$ is $\ell_i=L_i-E_i+1\geq 2S$ and $T_i=(E_i+L_i)/2$.

Example (3.3): let $W_1=[10,20]$ and $W_2=[25,50]$, $S_{12}=10$, $S=10$. Note that $\ell_1=11$ and $\ell_2=26$, W_2 is logical time window but W_1 is not. While if $W_1=[10,15]$ and $W_2=[16,24]$, $S_{12}=15$, $S=15$. Note that both W_1 and W_2 are not logical time windows, since if $t_1=E_1=10$, then $t_2<t_1+S_{12}=10+15=25>L_2=24$, that mean W_2 is not logical time definitely, not satisfies the separation constraint.

Case (1): Let $W_{i1},W_{i2},\ldots,W_{iN}$ are all disjoint logical time windows in this sequence s.t. $W_{i_k}\cap W_{i_j}=\phi, \forall i_k,i_j\in P$, $i_k\neq i_j$, then the optimal solution with cost Z=0 at $t_{i_1}=T_{i_1}<t_{i_2}=T_{i_2}<\cdots<t_{i_N}=T_{i_N}$ and $i_1\rightarrow i_2\rightarrow\ldots\rightarrow i_N$.

Proof: Without loosing the generality, let N=3 to show Z=0 and $1\rightarrow2\rightarrow3$. Since W_1,W_2 and ,W_3 are logical time windows this mean $S=\max\{S_{ij}\}$, $\forall i,j\in P$. Let $t_1=T_1$, $T_1+S\leq L_1<E_2<T_2$, then take:

$$t_2 = T_2 > T_1+S=t_1+S \qquad \ldots(a)$$

$\therefore$ $t_1=T_1$ and $t_2=T_2$ satisfy the window and separation conditions (WSC's). By applying relation (a) again for t_2 and t_3 we obtain that: $t_2=T_2$ and $t_3=T_3$ satisfy the WSCs.

$\therefore$ The optimal solution with cost Z=0 for N=3 and $1\rightarrow2\rightarrow3$.

Consequently, this case can be applied for N aircraft and for any sequence π.□

Case (2): Let $W=W_1=W_2=\ldots=W_N$ be the same large time window, then the optimal solution Z=0 at $t_{i_k}=T_{i_k}$ if T_{i_k} satisfies the separation constraint $\forall i_k\in P$ and $i_1\rightarrow i_2\rightarrow\ldots\rightarrow i_N$.

Proof: let's take any arbitrary sequence π. Since T_{i_k} satisfy the separation constraints, this means: $T_1 \leq T_2 - S_{12}$, $T_2 \leq T_3 - S_{23}, \ldots, T_{N-1} \leq T_N - S_{N-1,N}$. If we take $t_{i_k} = T_{i_k}$, then the landing times t_{i_k} satisfy the separation constraint $\forall i_k \in P$.

∴ The optimal solution with cost Z=0 and 1→2→…→N. □

Of course, this case can be applied for any sequence π.

Exercise (3.2): Find the SR for:

1. For N=5 let's have the following ALP information:

	P_1	P_2	P_3	P_4	P_5
E_i	129	111	123	89	96
T_i	155	123	135	98	106
L_i	191	135	147	110	118
g_i	10	30	30	30	30
h_i	10	30	30	30	30

S_{ij}	1	2	3	4	5
1	0	15	15	15	15
2	15	0	8	8	8
3	15	8	0	8	8
4	15	8	8	0	8
5	15	8	8	8	0

2. For N=5 let's have the following ALP information:

	P_1	P_2	P_3	P_4	P_5
E_i	146	241	90	95	108
T_i	155	250	93	98	111
L_i	164	259	96	101	114
g_i	10	10	30	30	30
h_i	10	10	30	30	30

S_{ij}	1	2	3	4	5
1	0	3	15	15	15
2	3	0	15	15	15
3	15	15	0	8	8
4	15	15	8	0	8
5	15	15	8	8	0

3. For N=5 let's have the following ALP information:

	P_1	P_2	P_3	P_4	P_5
E_i	241	146	108	90	95
T_i	250	155	111	93	98
L_i	259	164	114	96	101
g_i	10	10	30	30	30
h_i	10	10	30	30	30

S_{ij}	1	2	3	4	5
1	0	3	15	15	15
2	3	0	15	15	15
3	15	15	0	8	8
4	15	15	8	0	8
5	15	15	8	8	0