Combinatorial Optimization Problems

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Introduced By

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Chapter Three - 3

Aircraft Landing Problems

Solving ALP using Heuristic and CE Methods

Parallel Improving Technqiue

The order between aircraft (**sequencing the aircraft**) is setup according to priority rules which are based on the variables:

- **E_i**: The priority is given to the aircraft which has the sooner earliest landing time.
- **T_i** : The priority is given to the aircraft which has the earliest target landing time.
- **L_i** : The priority is given to the aircraft which has the earliest latest landing time.
- **E_i/g_i** : The priority is given to the aircraft which has the soonest earliest time.
- **L_i/h_i** : The priority is given to the aircraft which has the soonest latest time.
- **$Ti /(g_i+h_i)$**: The priority is given to the aircraft which has the soonest target.
- **$1/(g_i+h_i)$**: The priority is given to the aircraft which causes the most important advance and lateness penalty.

	P_1	P_2	P_3
E_i	129	195	89
T_i	155	258	98
L_i	559	744	510
g_i	10	10	30
h_i	10	10	30

	S_{ij}		
	1	2	3
1	0	3	15
2	3	0	15
3	15	15	0

Example : Let N=3

We have the following priority rules:

E_i: we have the sequence 3,1,2.

T_i: we have the sequence 3,1,2.

L_i: we have the sequence 3,1,2.

E_i/g_i=(12.9,19,5,2.97), we have the sequence 3,1,2.

L_i/h_i=(55.9,74.9,17), we have the sequence 3,1,2.

$T_i/(g_i+h_i)$=(7.75,12.9,1.63), we have the sequence 3,1,2.

$1/(g_i+h_i)$=(0.05,0.05,0.03), we have the sequence 3,1,2 or 3,2,1.

Solving ALP using Heuristic and CE Methods

Parallel Improving Technqiue

The adjusting landing time (scheduling aircraft)

Parallel Improving Algorithm (PIA)

Let P be the list of aircraft set up according to a priority rule and O={}.

1. $t_{P1} \leftarrow T_{P1}$; $P_1 \in O$.

2. **FOR** i = 2 : N

   $t_{Pi} \leftarrow \max(T_{Pi}, \max_{Pj \in O}(t_{Pj} + S_{Pi,Pj}))$

   END {FOR i}

3. **REPEAT**

   Calculate penalty Cost Z

   IF ($t_{Pi} > T_{Pi}$)

   Reduce the landing time by 1 unit of time

   ELSE { $t_{Pi} \leq T_{Pi}$}

   Increase the landing time by 1 unit of time

   END {IF}

   IF (the solution is unfeasible)

   Reject the change and keep the last feasible solution.

   BREAK.

   END {IF}

   UNTIL (there is increase of penalty cost)

Parallel Improving Technqiue

Example: For N=7

	P_1	P_2	P_3	P_4	P_5	P_6	P_7
E_i	129	195	89	96	110	120	124
T_i	155	258	98	106	123	135	138
L_i	559	744	510	521	555	576	577
g_i	10	10	30	30	30	30	30
h_i	10	10	30	30	30	30	30

S_{ij}	1	2	3	4	5	6	7
1	0	3	15	15	15	15	15
2	3	0	15	15	15	15	15
3	15	15	0	8	8	8	8
4	15	15	8	0	8	8	8
5	15	15	8	8	0	8	8
6	15	15	8	8	8	0	8
7	15	15	8	8	8	8	0

Suppose that the priority rule is the Ti. The order is as follows:

P_i	3	4	5	6	7	1	2

assign landing time to the 1st aircraft in the list (P_1=3) : $t_3 = T_3$=98, O={3}, then:

P_i	3	4	5	6	7	1	2	Z
t_i	98							0

For the 2nd aircraft in the list (P_2=4), then

$t_4 \leftarrow \max(T_4, \max_{P_j \in O}(t_3 + S_{3,4}))$=max(106,max(98+8))=106, O={3,4}.

P_i	3	4	5	6	7	1	2	Z
t_i	98	106						0

For the 3rd aircraft in the list (P_3=5), then

$t_5 \leftarrow \max(T_5, \max(t_3+S_{3,5}, t_4+S_{4,5}))$=max(123,max(98+8,106+8))=123, O={3,4,5}.

P_i	3	4	5	6	7	1	2	Z
t_i	98	106	123					0

Parallel Improving Technqiue

Continue example

For the 4th aircraft in the list (P_4=6), then t_6=max(135,max(98+8,106+8,123+8))=135, O={3,4,5,6}, Z=0.

P_i	3	4	5	6	7	1	2	Z
t_i	98	106	123	135				0

For the 5th aircraft in the list (P_5=7), then t_7=max(138,max(98+8,106+8,123+8,135+8))=143$\neq$138, O={3,4,5,6,7},

P_i	3	4	5	6	7	1	2	Z
t_i	98	106	123	135	143			150

here we need adjusting the landing time t_7=142, then t_6=134:

P_i	3	4	5	6	7	1	2	Z
t_i	98	106	123	134	142			150

And continue in decreasing until we obtain:

P_i	3	4	5	6	7	1	2	Z
t_i	98	106	123	131	139			150

If we continue another step we obtain:

P_i	3	4	5	6	7	1	2	Z
t_i	98	106	122	130	138			180

Since Z=180, we ignore this step and back to the last step when Z=150.

Parallel Improving Technqiue

Continue example

For the 6[th] aircraft in the list (P_6=1), then

T_1=max(155,max(98+15,106+15,123+15,131+15,139+15))=155,O={3,4,5,6,7,1}, now we need no adjusting the landing time so we obtain, Z=150

P_i	3	4	5	6	7	1	2	Z
t_i	98	106	123	131	139	155		150

For the 7th aircraft in the list (P7=2), then

T2=max(258,max(98+15,106+15,123+15,131+15,139+15,155+15))=258,O={3,4,5,6,7,1,2}, now we need no adjusting the landing time so we obtain, Z=150:

P_i	3	4	5	6	7	1	2	Z
t_i	98	106	123	131	139	155	258	150

This Table shows the implementation of PIA for this example.

Stage	**3**	**4**	**5**	**6**	**7**	**1**	**2**	**Cost Z**
1	98							0
2	98	106						0
3	98	106	123					0
4	98	106	123	135				0
5	98	106	123	131	139			150
6	98	106	118	131	139	155		150
7	98	106	118	131	139	155	258	150

Complete Enumeration Method (CEM)

When using CEM, in sequencing stage we will try all the possible permutation of N planes which equal to N!, while in scheduling stage we will apply two methods:

- **Exhaustive Search Method (ESM** we try all possibilities starting from E_i ending in L_i. The total number of all possibilities for scheduling is $\prod_{i=1}^{N}(L_i - E_i + 1)$
- **PIA.**

the total complexity (C(N)) for sequencing and scheduling using CEM is:

$$C(N)=N!*\prod_{i=1}^{N}(L_i - E_i + 1) \qquad \ldots(8)$$

For $E_i=T_i=L_i$, (Z=0) $\forall i \in P$, then C(N)=N!.

Remark: In general, if :

R: the number of pairs of aircraft which are satisfy SR's.

D: the number of pairs of aircraft which are not submitted to SR's, represented by the variables δ_{ij} in matrix A.

$R+D=C_2^N=N*(N-1)/2$, for the ALP we have 2^D sequences can be try to find the best sequence. In some ALP, N! may be larger than 2^D and vice versa.

N	C(N)	N!	C_2^N	R	D	2^D
8	9.689287×10^{16}	40320	28	17	11	2048*
9	2.486859×10^{20}	362880*	36	17	19	524288
10	1.892271×10^{23}	3628800*	45	23	22	4194304
15	5.084773×10^{41}	1.3077×10^{12}*	105	44	61	2.305843×10^{18}

Complete Enumeration Method (CEM)

Example : N=3

	P_1	P_2	P_3
E_i	130	127	96
T_i	131	128	97
L_i	133	130	99
g_i	10	10	30
h_i	10	10	30

	S_{ij}		
	1	2	3
1	0	4	4
2	4	0	4
3	4	4	0

CEM-ESM

The general Complexity is C(3)=6*64=384.The number of SR=3, R=3 and D=0 so we have the unique sequence π=(3,2,1), then C(3) reduces to 64 possible. Then the best solutions using CEM-ESM are:

1 - 96,127,131, Z=40.

2 - 96,128,132, Z=40.

3 - 97,127,131, Z=10.

4 - 97,128,132, Z=10.

CEM-PIA

we have the unique sequence π=(3,2,1), then the best solution using CEM-PIA is:

1 - 97,127,131, Z=10.

Data Set

Table (1)

	P_1	P_2	P_3
E_i	129	195	89
T_i	155	258	98
L_i	559	744	510
g_i	10	10	30
h_i	10	10	30

	S_{ij}		
	1	2	3
1	0	3	15
2	3	0	15
3	15	15	0

Table(2-1)

	P_1	P_2	P_3	P_4	P_5	P_6	P_7	P_8	P_9	P_{10}
E_i	129	195	89	96	110	120	124	126	135	160
T_i	155	258	98	106	123	135	138	140	150	180
L_i	559	744	510	521	555	576	577	573	591	657
g_i	10	10	30	30	30	30	30	30	30	30
h_i	10	10	30	30	30	30	30	30	30	30

Table(2-2)

S_{ij}	1	2	3	4	5	6	7	8	9	10
1	0	3	15	15	15	15	15	15	15	15
2	3	0	15	15	15	15	15	15	15	15
3	15	15	0	8	8	8	8	8	8	8
4	15	15	8	0	8	8	8	8	8	8
5	15	15	8	8	0	8	8	8	8	8
6	15	15	8	8	8	0	8	8	8	8
7	15	15	8	8	8	8	0	8	8	8
8	15	15	8	8	8	8	8	0	8	8
9	15	15	8	8	8	8	8	8	0	8
10	15	15	8	8	8	8	8	8	8	0

Exercises

1. Calculate the TWT for:

- from Table (1), Z_{UB}=900.
- Table(2-1) and table (2-2), for N=10, for 1st 5 aircraft, Z_{UB}=90.

2. Find the SR for N=5

	P_1	P_2	P_3	P_4	P_5
E_i	129	111	123	89	96
T_i	155	123	135	98	106
L_i	191	135	147	110	118
g_i	10	30	30	30	30
h_i	10	30	30	30	30

S_{ij}	1	2	3	4	5
1	0	15	15	15	15
2	15	0	8	8	8
3	15	8	0	8	8
4	15	8	8	0	8
5	15	8	8	8	0

	P_1	P_2	P_3	P_4	P_5
E_i	146	241	90	95	108
T_i	155	250	93	98	111
L_i	164	259	96	101	114
g_i	10	10	30	30	30
h_i	10	10	30	30	30

S_{ij}	1	2	3	4	5
1	0	3	15	15	15
2	3	0	15	15	15
3	15	15	0	8	8
4	15	15	8	0	8
5	15	15	8	8	0

3. Find the priority rules for Exercise (2).

Exercises

4. Apply PIA using T_i priority for N=5

	P_1	P_2	P_3	P_4	P_5
E_i	129	190	84	89	100
T_i	155	250	93	98	111
L_i	305	400	143	148	161
g_i	10	30	30	30	30
h_i	10	30	30	30	30

S_{ij}	1	2	3	4	5
1	0	3	15	15	15
2	3	0	15	15	15
3	15	15	0	8	8
4	15	15	8	0	8
5	15	15	8	8	0

	P_1	P_2	P_3	P_4	P_5
E_i	146	249	95	103	120
T_i	155	258	98	106	123
L_i	164	267	101	109	126
g_i	10	30	30	30	30
h_i	10	30	30	30	30

S_{ij}	1	2	3	4	5
1	0	3	15	15	15
2	3	0	15	15	15
3	15	15	0	8	8
4	15	15	8	0	8
5	15	15	8	8	0

5. Find C(N), The number of SR, R, D , the possible sequences π, then find the optimal solution for the following ALP using CEM-ESM and CEM-PIA

	P_1	P_2	P_3
E_i	130	127	96
T_i	131	128	97
L_i	132	129	98
g_i	10	10	30
h_i	10	10	30

	S_{ij}		
	1	2	3
1	0	2	2
2	2	0	2
3	2	2	0